A.4 Modal analysis

Modal analysis is a very useful tool for linking experimental analysis with mathematical modelling. It is particularly convenient for predicting the dynamic interaction of interconnected substructures.

When using experimental mobility data, modal analysis uses statistical methods to extract the modal parameters, including natural frequencies, damping and modal mass (or stiffness), within the frequency range of interest. Several different methods are available for identifying these parameters accurately and taking into account the effect of modes outside the frequency range of interest.

When using mathematical models, the modal parameters can be extracted from the computed mass, stiffness and damping matrices of the substructures by eigenvalue and eigenvector computation or other matrix reduction procedures. These procedures are often more efficient that the direct inversion of the entire impedance matrix.

© ISO 2011 – All rights reserved

Annex B
(informative)

Mobility as a frequency-response function

B.1 Harmonic excitation

When a time function can be expressed as

$$\boldsymbol{x}(t) = A \cos(\omega t + \varphi) \tag{B.1}$$

where

A is the amplitude of the harmonic waveform;

ω is the circular frequency;

t is the time;

φ is an initial phase angle;

then the function can be represented on a complex plane as

$$\boldsymbol{x}(t) = A \exp[\mathrm{j}(\omega t + \varphi)] \tag{B.2}$$

where

$\mathrm{j} = \sqrt{-1}$

$x(t)$ is the projection of $\boldsymbol{x}(t)$ on the real axis.

The function $\boldsymbol{x}(t)$ can be thought of as a rotating vector centred at the origin of the complex plane. This rotating vector is commonly called a phasor[1]. If the ratio of two phasors is formed at a given frequency, the result is a complex number, and is not a function of time. If a set of these complex numbers is formed for all frequencies of interest, the result is called a frequency-response function. If the two phasors were

$$\boldsymbol{x}_1(\omega,t) = A_1(\omega) \exp\{\mathrm{j}[\omega t + \varphi_1(\omega)]\}$$

$$\boldsymbol{x}_2(\omega,t) = A_2(\omega) \exp\{\mathrm{j}[\omega t + \varphi_2(\omega)]\}$$

then

$$H(\omega) = \frac{\boldsymbol{x}_1}{\boldsymbol{x}_2} = B(\omega) \exp[\mathrm{j}\theta(\omega)] \tag{B.3}$$

where

$B(\omega) = A_1(\omega)/A_2(\omega)$

$\theta(\omega) = \varphi_1(\omega) - \varphi_2(\omega)$

---

1) Some authors define the phasor as $A \exp(\mathrm{j}\varphi)$. Both definitions are consistent with the use of the term phasor in this part of ISO 7626, which is only concerned with the ratio of phasors.

© ISO 2011 – All rights reserved

Both $B(\omega)$ and $\theta(\omega)$ are functions of the circular frequency, ω. The frequency-response function, $H(\omega)$, can be expressed in polar form by the amplitude and phase, $B(\omega)$ and $\theta(\omega)$, or in rectangular form as real and imaginary parts:

$$H(\omega) = R(\omega) + \mathrm{j}\, I(\omega) \tag{B.4}$$

where

$$R(\omega) = B(\omega) \cos \theta(\omega)$$

$$I(\omega) = B(\omega) \sin \theta(\omega)$$

Mobility can be considered as a frequency-response function given by the ratio of the velocity and force phasors. Similar arguments can be made for the other mobility-like quantities.

B.2 Random excitation

In random vibration with a stationary Gaussian distribution, a fundamental equation that relates the output and input of a linear bilateral system is

$$G_{12}(\omega) = H(\omega)\, G_{22}(\omega) \tag{B.5}$$

where

$G_{12}(\omega)$ is the cross-spectral density between the input and output of the system;

$G_{22}(\omega)$ is the auto-spectral density of the input;

$H(\omega)$ is the frequency-response function of the system.

G_{12} is a complex function of frequency and G_{22} is a real function of frequency; hence, $H(\omega)$ is a complex function of frequency. Mobility can be considered as a frequency-response function given by the ratio of the cross-spectral density between the velocity and force excitation to the auto-spectral density of the force excitation, the input. In actual practice, only estimates of the cross and auto spectra are available. Hence, mobilities can only be estimated using a random excitation. The estimation errors can be made less than other measurement errors and do not necessarily limit the accuracy of the measurements.

B.3 Transient excitation

In transient vibration, the input and response of a linear system can be related through the expression

$$X_1(\omega) = H(\omega)\, X_2(\omega) \tag{B.6}$$

where

$X_1(\omega)$ is the Fourier transform of the output $x_1(t)$;

$X_2(\omega)$ is the Fourier transform of the input $x_2(t)$;

$H(\omega)$ is the frequency-response function of the system.

All of the quantities in Equation (B.6) are complex functions of frequency. The inverse Fourier transform of $H(\omega)$ is the unit impulse response function, $h(t)$, of the system; that is, if a unit impulse is applied to the input, the system output is given by $h(t)$. Mobility can be considered as a frequency-response function given by the ratio of the Fourier transforms of the time histories of the velocity response and of the input force.

In actual practice, the discrete Fourier transform (DFT) is used as an approximation of the continuous Fourier transform. Careful selection of sample rate and sample size can reduce the errors of this approximation to levels less than other measurement errors. The use of the DFT does not, therefore, necessarily limit the accuracy of the measurements.

© ISO 2011 – All rights reserved

Annex C
(informative)

Determination of impedance head attachment compliance and damping

Overall calibration and determination of the sensitivity of a mobility measurement set-up, including the attachment hardware between the vibration exciter and the structure under test, may be accomplished using the following test procedure. This procedure also provides data on the attachment compliance and the structural damping of the impedance head (see Reference [13]).

The test shall be carried out by mounting a large rigid block on a sufficiently compliant support so that the natural frequency of the block on the compliant support is 2 Hz or less. A cylindrical block of sufficient mass should be used to place the anti-resonance (response dip) frequency well within the frequency range of interest. The magnitude, in kilograms, of the appropriate mass of the block, m, may be estimated from

$$m = 1/[(2\pi f)^2 C] \tag{C.1}$$

where

f is a convenient frequency, in hertz, in the upper part of the frequency range over which mechanical mobility measurements are to be carried out;

C is an estimate of the compliance, in metres per newton, of the impedance head and the attachment hardware between the force transducer and the structure under test.

In order to design the compliant support for the calibration block, Equation (C.2) yields the minimum required support compliance, C_s, for the block of mass m:

$$C_s = 1/[(2\pi f_s)^2 m] \tag{C.2}$$

where

f_s is the natural frequency, in hertz, of the block on the compliant support (2 Hz or lower);

m is the mass, in kilograms, of the block selected for the test.

The impedance head (or the force transducer) should be mounted colinear with the centre of gravity of the calibration block. The transducer manufacturer's specifications for mounting bolt torque should be closely adhered to. The mounting hardware should be as closely as possible identical to that to be used when measuring on the structure. The vibration exciter should be attached to the impedance head (or to the force transducer) in the manner to be used in the mobility measurement. The vibration exciter should be supplied with the same type of excitation waveform which is to be used during the mobility measurement. The range of frequencies to be used for the test with the calibration block should extend beyond the frequency range of interest at the high end.

NOTE 1 The mathematical model for this calibration test set-up is given in Reference [7].

The output signals from the force transducer and the motion response transducers should be processed in the same way as for the mobility measurement and should be documented (e.g. plotted as either a mobility or accelerance curve on suitable graph paper).

Figure C.1 shows a typical mobility data curve. The results shown in Figure C.1 were obtained with a calibration block having a mass of 5,5 kg and represent a rather typical attachment compliance of the impedance head of about 8×10^{-10} m/N. In the frequency range from 30 Hz to 1 100 Hz, the measured mobility curve closely approaches that of the mass of the block used for the calibration, plus the mass of the impedance head below the force transducer, combined with the mass of the attachment hardware. If the mass indicated by the low-frequency portion of the results of the mobility measurement does not equal the total calibration system mass, the mobility measuring system has failings which should be corrected. Inadequate sensitivity, incorrect calibration or electronic instrument problems are likely causes.

© ISO 2011 – All rights reserved

The anti-resonance (response dip) in the measured mobility curve at 2 400 Hz indicates that, at that frequency, the mobility of the calibration block has the same magnitude as that due to the attachment compliance of the impedance head. Since the two mobilities have essentially opposite phase, the total mobility magnitude tends towards zero at the anti-resonance frequency. It remains finite because of damping at the interface between the impedance head and the calibration block. This also accounts for the 0° phase angle at the anti-resonance.

At frequencies above the 2 400 Hz anti-resonance, the measured mobility curve approaches asymptotically the series combination of the compliance of the mounting hardware and that of the impedance head itself. It may be difficult to check the total compliance if the attachment has considerable damping. If the anti-resonance dip is not very sharp, the mobility curve does not approach the effective compliance asymptote at high frequencies.

NOTE 2 The irregularity at 4 600 Hz in the measured mobility curve shown in Figure C.1 is the result of a transverse rocking mode of the impedance head about its attachment point.

If the overall calibration and sensitivity test is extended to sufficiently high frequencies, the mobility curve becomes controlled by the effective damping of the impedance head and its mounting hardware. The magnitude of the effective damping can seldom be computed. Thus, the test described in this annex is the only practical way to determine how much damping is present in the coupling of the force transducer to the structure to be tested.

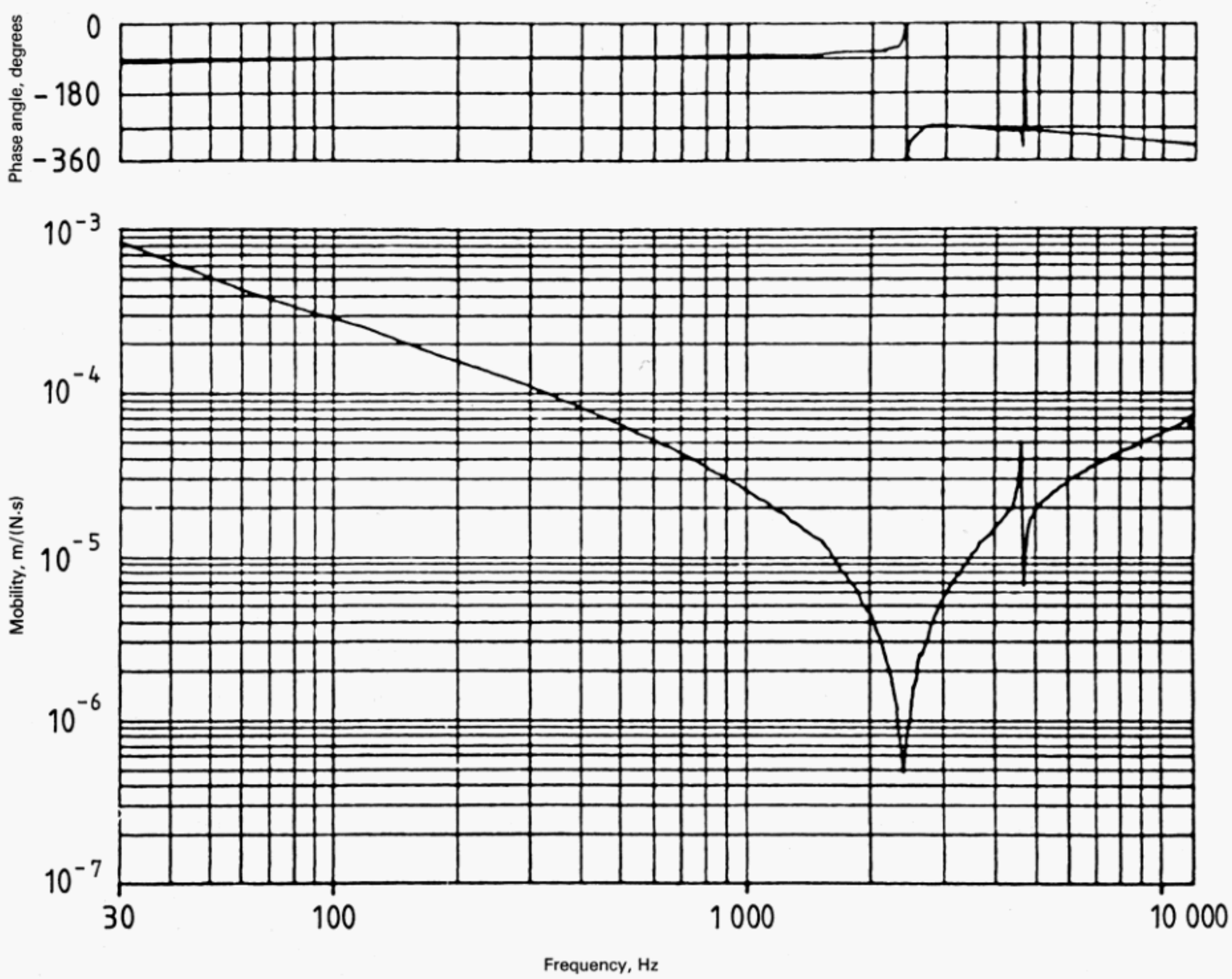

Figure C.1 — Example of test results showing the effect of impedance head attachment compliance (256 averages)

© ISO 2011 – All rights reserved

Bibliography

[1] ISO 5347-18, *Methods for the calibration of vibration and shock pick-ups — Part 18: Testing of transient temperature sensitivity*

[2] ISO 5348, *Mechanical vibration and shock — Mechanical mounting of accelerometers*

[3] ISO 10846 (all parts), *Acoustics and vibration — Laboratory measurement of vibro-acoustic transfer properties of resilient elements*

[4] O'HARA, G.J. Mechanical impedance and mobility concepts. *J. Acoust. Soc. Am.* 1967, **41**, pp. 1180-1184

[5] EWINS, D.J. Measurement and application of mechanical impedance data: Part 1 — Introduction and ground rules; Part 2 — Measurement techniques; Part 3 — Interpretation and application of measured data. *J. Soc. Environ. Eng.* 1975, **14**(3), pp. 3-13; 1976, **15**(1), pp. 22-33; 1976, **15**(2), pp. 7-17

[6] SAINSBURY, M.G., EWINS, D.J. Vibration analysis of a damped machinery foundation structure using the dynamic stiffness coupling technique. *Trans. ASME — J. Eng. Ind.*1974, **96**, pp. 1000-1005

[7] HIXSON, E.J. Mechanical impedance. In: PIERSOL, A.G., PAEZ, T.L., editors. *Harris' shock and vibration handbook*, 6th edition, Chapter 10. New York, NY: McGraw-Hill, 2009

[8] RUBIN, S. Mechanical immittance- and transmission-matrix concepts, *J. Acoust. Soc. Am.* 1967, **41**, pp. 1171-1179

[9] EWINS, D.J. Whys and wherefores of modal testing. *J. Soc. Environ. Eng.*1979, **18**(3), pp. 3-15

[10] EWINS, D.J., GRIFFIN, J. A state-of-the-art assessment of mobility measurement techniques — Results for the mid-range structures (30–3000 Hz). *J. Sound Vib.* 1981, **78**, pp. 197-222

[11] EWINS, D.J. State-of-the-art assessment of mobility measurements — A summary of European results. *Shock Vib. Bull.* 1981, **51**, pp. 15-35

[12] EWINS, D.J. State-of-the-art assessment of mobility measurement techniques (SAMM) — Summary of results. *J. Soc. Environ. Eng.* 1981, **20**, pp. 3-13

[13] BROWNJOHN, J.M.W., STEELE, G.H., CAWLEY, P., ADAMS, R.D. Errors in mechanical impedance data obtained with impedance heads. *J. Sound Vib.* 1980, **73**, pp. 461-468

[14] KLAASSEN, K.B. Piezoelectric accelerometers. In: REGTIEN, P.P.L,, editor, *Modern electronic measuring systems*, chap. III. Delft: Delft University Press, 1978

[15] BOUCHE, R.K. *Calibration of shock and vibration measuring transducers*. Shock and Vibration Information Center, U.S. Department of Defense, 1979. (Monograph SVM-11.)

[16] CANSDALE, R., GAUKROGER, D.R., SKINGLE, C.W. *A technique for measuring impedances of a spinning model rotor*. Farnborough: Royal Aircraft Establishment, 1971. (Technical Report TR 71092.)

[17] KUMME, R. A new calibration facility for dynamic forces up to 10 kN. In: *Proc. 17th IMEKO World Congress*, pp. 305-308, 2003

[18] ISO 7626-2, *Vibration and shock — Experimental determination of mechanical mobility — Part 2: Measurements using single-point translation excitation with an attached vibration exciter*

© ISO 2011 – All rights reserved

---,''',',''',''',-,-,,''',,',,',,''',,',,''',,',''''',,,,,',--

---,''',',''',''',-,-,,''',,',,',,''',,',,''',,',''''',,,,,',--