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IEEE Standard Jitter and Phase Noise

IEEE Instrumentation and Measurement Society

Developed by the
Waveform Generation, Measurement, and Analysis Technical Committee

IEEE Std 2414™-2020

STANDARDS

IEEE

IEEE Standard for Jitter and Phase Noise

Developed by

Waveform Generation, Measurement, and Analysis Technical Committee
of the
IEEE Instrumentation and Measurement Society

Approved 24 September 2020

IEEE SA Standards Board

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Introduction

This introduction is not part of IEEE Std 2414-2020, IEEE Standard for Jitter and Phase Noise.

Jitter is one of the most important specifications when measuring the performance of several types of electronic components and/or systems. That figure of merit has, in fact, a relevant impact on the design, operation, and verification of many of today products and services. At the component/systemic level, jitter measurements can lead to malfunction causes and to effectively diminish their deleterious effect on the overall system performance.

This standard defines the terms, definitions, and mathematical models used to specify and characterize jitter. It is intended for the following:

- Individuals and organizations who specify electronic devices, equipment, and systems subject to jitter to be purchased
- Individuals and organizations who purchase electronic devices, equipment, and systems subject to jitter to be applied in their products
- Individuals and organizations whose responsibility is to characterize and write reports on jitter for specific applications
- Suppliers interested in providing high-quality and high-performance electronic components devices, equipment, and systems to acquirers

This standard is designed to help organizations and individuals incorporate quality considerations during the definition, evaluation, selection, and acceptance of supplier of electronic devices for operational use in their equipment.

This standard is intended to satisfy the following objectives:

- Promote consistency within organizations in acquiring third-party electronic devices, equipment, and systems from suppliers
- Provide useful practices on including quality considerations during acquisition planning
- Provide useful practices on evaluating and qualifying supplier capabilities to meet user requirements
- Assist individuals and organizations judging the quality and suitability of devices, equipment, and systems for referral to end users

Several standards have previously been written that address the definition and modeling of jitter. These include the following standards:[1]

- IEEE Std 1139-2008, IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology—Random Instabilities [B7]

[1] The numbers in brackets correspond to those of the bibliography in Annex B.

- IEEE Std 802.3-2008, IEEE Standard for Information technology—Telecommunications and information exchange between systems—Local and metropolitan area networks—Specific requirements—Part 3: Carrier Sense Multiple Access with Collision Detection (CSMA/CD) Access Method and Physical Layer Specifications [B7]
- IEC 60469:2013, Transactions, pulses and related waveforms – Terms, definitions and algorithms [B9]
- IEC 60679-6:2011, Quartz crystal-controlled oscillators of assessed quality – Part 6: Phase jitter measurement method for quartz crystal oscillators and SAW oscillators – Application guidelines [B10]
- IEC 61280-2-3:2009, Fibre optic communication subsystem test procedures – Part 2-3: Digital systems – Jitter and wander measurements [B11]
- ITU-R Recommendation BT.1363, Jitter Specifications and Methods for Jitter Measurements of Bit-Serial Signals Conforming to Recommendations ITU-R BT.656, BT.656, ITU-R BT.799 and ITU-R BT.1120, 1998 [B12]
- ITU-T G.810, SERIES G: Transmission Systems and Media Digital Transmission Systems, Digital networks – Design objectives for digital networks – Definitions and terminology for synchronization networks, 1996 [B13]

The main aim of this standard is the harmonization of terminology and models included in these standards.

Contents

IEEE Standard for Jitter and Phase Noise

1. Overview

1.1 Scope

The standard defines specifications, modeling methods and terminology for the dispersion of specified instants of repetitive and/or periodic signals in electronics, telecommunications and measurement, which is referred to as jitter and phase noise.

1.2 Purpose

The purpose of the standard is to facilitate accurate and precise communication concerning jitter and phase noise and the models for measuring them. Because of the broad applicability of such terms in the electronics industries (such as computer, telecommunication, and measurement instrumentation industries), developing unambiguous definitions and the presentation of models for their measurement is important for communication between manufacturers, users and consumers.

2. Definitions, symbols, acronyms and abbreviations

2.1 Definitions

For the purposes of this document, the following terms and definitions apply. The *IEEE Standards Dictionary Online* should be consulted for terms not defined in this clause. [1]

amplitude of the timing jitter: The range of deviations of the actual reference instants of a timing waveform from their ideal values.

bounded uncorrelated jitter (BUJ): The non-periodic contribution to the deterministic jitter in a two-level (binary) or multilevel signal that is associated with signal transitions but which shows no correlation to the signal itself.

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[1] *IEEE Standards Dictionary Online* is available at: http://dictionary.ieee.org. An IEEE Account is required for access to the dictionary, and one can be created at no charge on the dictionary sign-in page.

cycle-to-cycle jitter (C2C): The difference between consecutive period durations in an ideally periodic signal.

data-dependent jitter (DDJ): The contribution to the deterministic jitter in a two-level (binary) or multilevel signal, typically referenced to the transitions among the states of the signal.

deterministic jitter (DJ): The contribution to the jitter in which successive reference instants are deterministically predicted.

duty cycle distortion (DCD): The variable delay between the positive-going and the negative-going reference level transitions between cycles of the signal of interest.

event: Any feature in a waveform whose instant of occurrence is uniquely defined.

frequency drift rate: A measure of the time rate of change of the frequency offset (i.e. frequency stability).

frequency offset: A measure of the difference between the observed clock frequency and its ideal value.

instant: A particular time value within a waveform epoch that, unless otherwise specified, is referenced relative to the initial instant of that waveform epoch.

jitter: The deviation of a sequence of reference instants from their ideal values.

NOTE—The reference level instant used in this standard is the user-defined reference instant epoch.

maximum time interval error (MTIE): The maximum TIE (in magnitude) within a user-defined observation interval.

peak-to-peak jitter: The difference between the maximum and the minimum values of either the TE or the TIE.

periodic jitter (PJ): The periodic contribution to the deterministic jitter in a two-level (binary) or multilevel signal, typically referenced to the transitions among the states of the signal.

period jitter (PEJ): The jitter in the period of a repetitive signal or its waveform.

phase noise: random fluctuations in the phase of a periodic signal.

random jitter (RJ): The contribution to the jitter in which successive reference instants cannot be deterministically predicted and, thus, must be described in statistical terms.

reference instant: The instant of the occurrence of a specified event.

rms jitter: The root-mean-square value of either the TE or the TIE.

signal: A physical phenomenon that is a function of time and space epoch.

time interval error (TIE): The difference between two specified time intervals, each delimited by two event occurrences; the first interval is in the actual waveform and the other is in the ideal waveform.

timing deviation (TDEV): The deviation of the actual reference instants from the ideal reference instants for a given event and for a given number of repeats of this event.

timing error (TE): The difference between the actual reference instant of an event and its ideal value.

timing jitter: The deviation of the actual reference instants associated with a timing waveform with respect to their ideal values.

total jitter (TJ): The sum of the random jitter and the deterministic jitter.

transition density (D): the probability that two consecutive bits are different.

unit interval (UI): The shortest duration between the ideal reference instants of two subsequent event occurrences.

wander: That part of RJ with spectral density below 10 Hz.

waveform: A representation of a signal (for example, a graph, plot, oscilloscope presentation, discrete time series, equations, or table of values). This term refers to a measured or otherwise-defined estimate of the physical phenomenon or signal.

waveform epoch: An interval to which consideration of a waveform is restricted for a particular calculation, procedure, or discussion. Except when otherwise specified, it is assumed to be the span over which the waveform is measured or defined.

waveform feature (feature): A specified portion or segment of a waveform.

2.2 Acronyms and abbreviations

BUJ	bounded uncorrelated Jitter
C2C	cycle-to-cycle jitter
D	*transition density* in the digital waveform encoding the bit sequence, i.e. the probability that two consecutive bits are different
DCD	duty cycle distortion
DDJ	data-dependent jitter
DJ	deterministic jitter
ISI	inter symbol interference
MTIE	maximum time interval error
PDF	probability density function
PJ	periodic jitter
PLL	phase-locked loop
PEJ	period jitter
PJ	periodic jitter
PSD	power spectral density
RJ	random jitter

TDEV	timing deviation
TE	timing error
TIE	time interval error
TJ	total jitter
UI	unit interval

2.3 Symbols

$J_{TE,n}$	timing error (TE)
t_n	*n*th sample instant of the actual timing waveform
$t_{id,n}$	*n*th sample instant of the ideal timing-signal waveform
T_{id}	ideal period
$TDEV[L,N]$	timing deviation
$J_{TIE,n,L}$	time interval error (TIE)
$TIE_{\Sigma}[n,L]$	accumulated TIE
T_n	*n*th observed period
$MTIE[n_1,n_2,L]$	maximum time interval error
PEJ_n	period jitter
$C2C_n$	cycle-to-cycle jitter
$f_{RJ}(x)$	random jitter probability density function
DCD_n	duty cycle distortion
$f_x(x)$	probability density function of the random variable X
$\delta(\bullet)$	Dirac pulse function
BUJ_{pk}	BUJ peak jitter value
σ_{BUJ}	standard deviation of the BUJ distribution
C_{BUJ}	BUJ normalization coefficient
TJ_{rms}	RMS jitter
TJ_{pp}	peak-to-peak jitter
UI_{rms}	units for RMS jitter measurements
UI_{pp}	units for peak-to-peak jitter measurements
BER	bit error rate
D	*transition density* in the digital waveform encoding the bit sequence, i.e., the probability that two consecutive bits are different
$\Phi(z)$	Normal standard cumulative function
$v_{id}(t)$	ideal sinusoidal signal
$v(t)$	non-ideal sinusoidal signal
A_0	sinusoidal ideal peak amplitude
$\varepsilon(t)$	deviation from the ideal amplitude
f_0	ideal frequency

$\varphi(t)$	random phase deviation
$\mathcal{L}(f)$	single-sideband phase noise to carrier ratio
φ_{rms}	RMS phase noise over a frequency range
$J_{\varphi,RMS}$	RMS phase jitter expressed in time units
$S_v(f)$	one sided PSD of the signal v

3. Jitter models and figures of merit

3.1 General concepts

Jitter is a general term that describes a phenomenon consisting in the deviation of the reference instants of a sequence of events from their ideal values. Jitter may be interpreted in several ways, as will be discussed in this standard, depending on the practical effect of interest and its application.

Jitter has many different physical sources. However, from a behavioral point of view, jitter can be modeled by means of statistical distributions, time trends, and frequency domain analysis of event reference instant deviations.

Jitter appears either on ideally periodic signals, like those generated by clock sources and sine wave generators; inherently non-periodic signals, like baseband digital signals; or in general repetitive signals that include asynchronous events.

It is the features and instants in a waveform that are amenable to measurement and analysis and, hence, the reference to waveform characteristics in this standard. The word "actual" when it refers to a waveform or its features is used to indicate the measured or acquired waveform or its features.

3.2 Timing jitter

When the jitter specifically affects signals that are intended to provide a timing reference, such as clock signals or sync signals, it is called timing jitter. The following subclauses provide specific mathematical symbols and models. It is worth noticing that when the effect is due to long-term phenomena, the term wander is used instead. The boundary between short-term and long-term phenomena is conventionally placed, in this context, at 10 Hz in the power spectral density (see 3.5.1 for details).

Timing jitter is the deviation of the actual reference instants associated with a timing waveform with respect to their ideal values.

Timing jitter provides a measure of the short-term instability of the timing circuit. Timing jitter is characterized by means of its spectral properties, its statistical distribution in time or its time trend. In these two last cases, timing jitter shall be expressed in units of time or in unit interval (UI).

The amplitude of the timing jitter is defined as the range of deviations of the actual reference instants of a timing waveform from their ideal values.

The example shown in Figure 1 presents a case of timing jitter induced by additive amplitude noise on the timing signal. In this figure, the noisy positive-going transition of the waveform crosses the detection threshold over a range of reference instants that deviate from the ideal reference instant. Note that only one threshold crossing is detected per repetition of the transition.

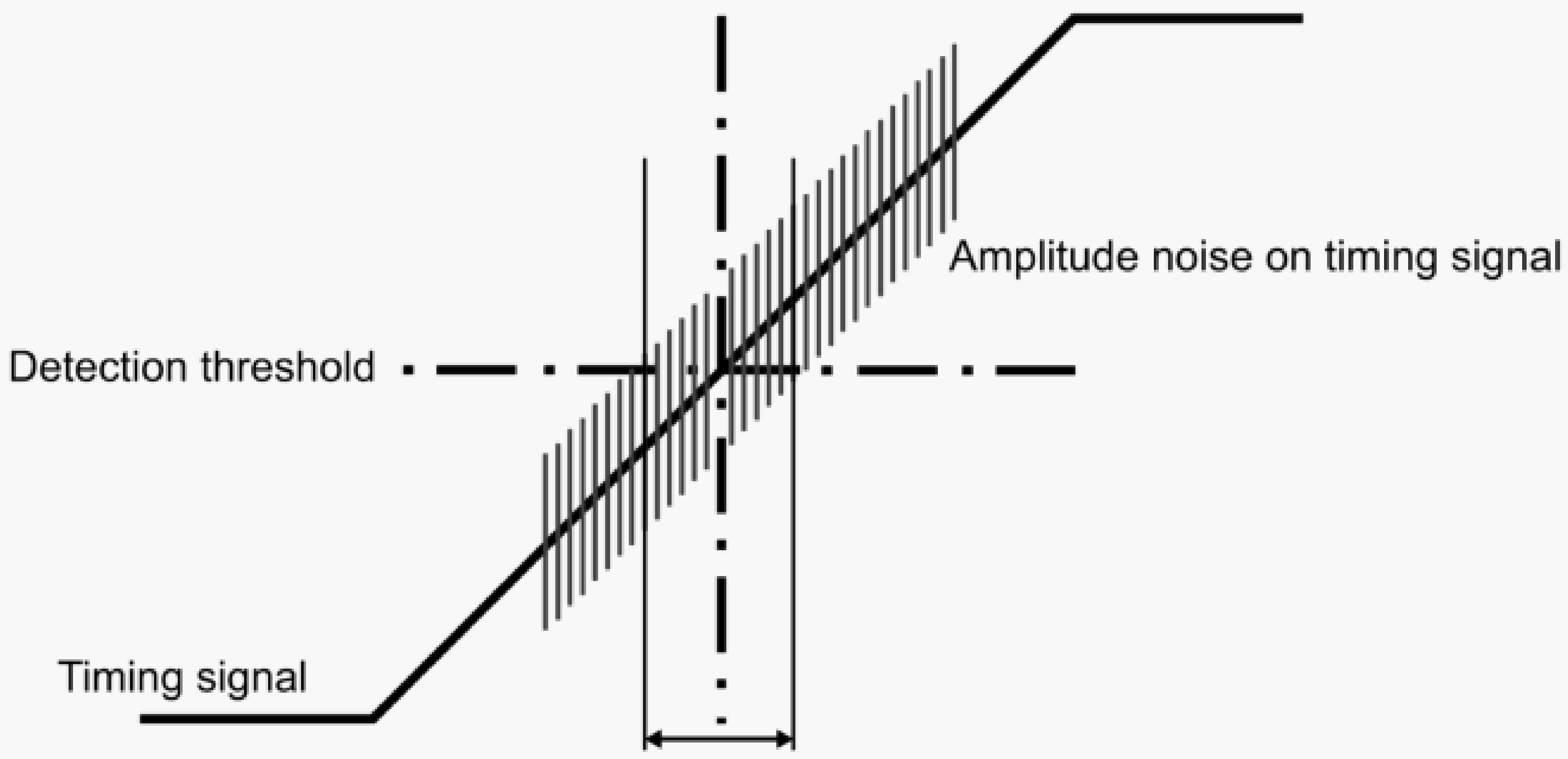

Figure 1—Example of timing jitter induced by additive amplitude noise

3.2.1 Timing error

The timing error (TE) is the difference between the actual reference instant of an event and its ideal value, and is given by Equation (1):

$$J_{TE,n} = t_n - t_{id,n} \tag{1}$$

where

t_n	is the reference instant of the *n*th occurrence of an event
$t_{id,n}$	is the ideal reference instant of the *n*th occurrence of the same even
$J_{TE,n}$	is the timing error

For example, timing error quantifies the delay or advance of the instant of the actual transition of a signal compared to its ideal transition instant (IEC 61280-2-3:2009 [B11]). To measure $J_{TE,n}$, $t_{id,n}$ must be either known or estimated (EN 60679-1:2007-06 (P9450) [B3]). Therefore, the estimation of $J_{TE,n}$ requires the comparison of t_n with the $t_{id,n}$ for a reference clock (Figure 2). The reference clock synchronization and/or strobe signal is transmitted or not. In the latter case, a clock and data recovery is required (Teledyne Lacroy (2014) [B31]).

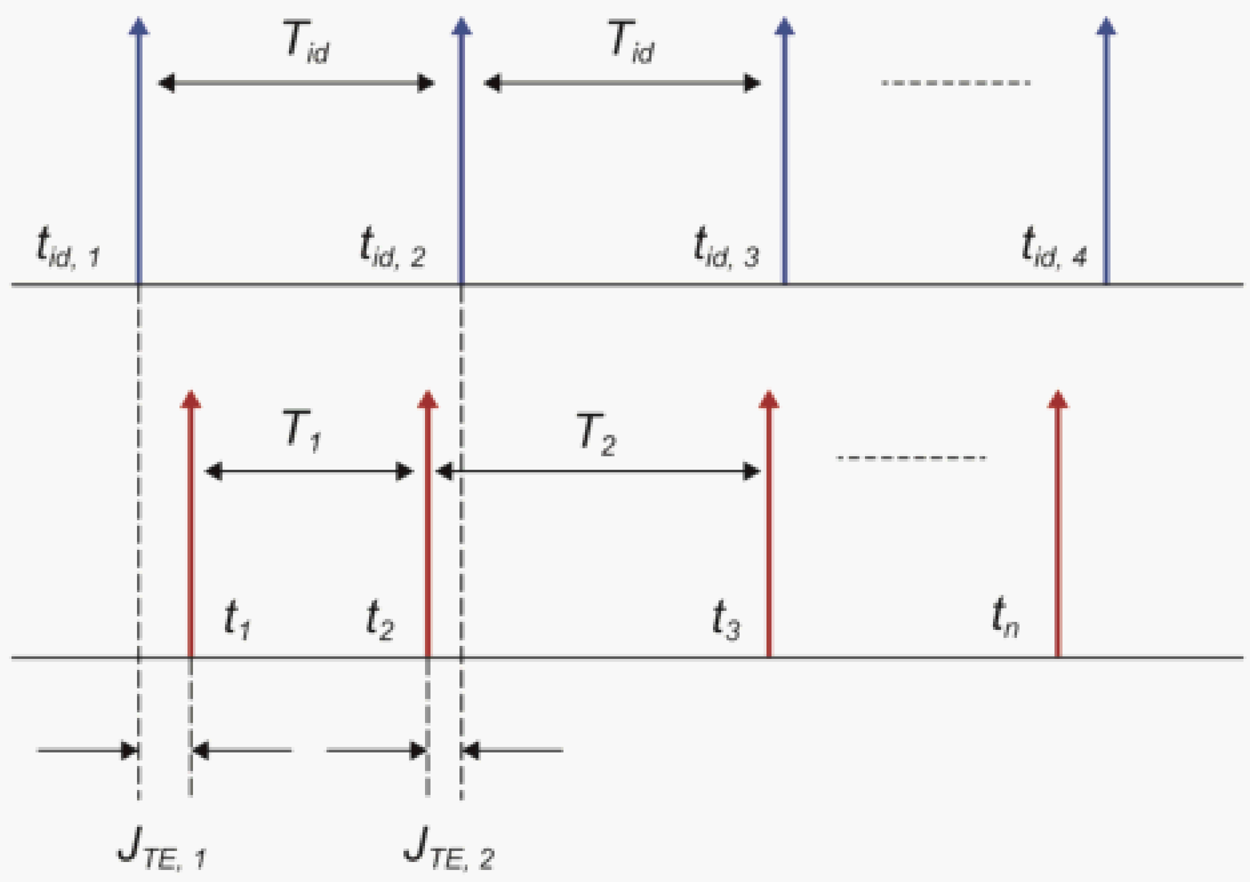

Figure 2—Timing error example for a clock signal (from DSC AN-1 [B2]), where T_{id} is the ideal period

3.2.1.1 Timing deviation

The timing deviation (TDEV) provides an estimate of the deviation of the actual reference instants from the ideal reference instants for a given periodic event. Given a total number of N repetitions of the event, a subset is extracted by taking one occurrence out of every L. TDEV estimates the variability of J_{TE} versus L by means of Equation (2) (from ITU-T G.810, SERIES G [B13]):

$$TDEV[L,N]=\sqrt{\frac{1}{6L^2(N-3L+1)}\left[\sum_{i=j}^{L+j+1}\left(J_{TE,i+2L}-J_{TE,i+L}+J_{TE,i}\right)\right]^2}, \quad L=1,2,\ldots,\left\lfloor\frac{N}{3}\right\rfloor \tag{2}$$

where

$\lfloor x \rfloor$ is a function returning the largest integer value less than or equal to x

TDEV also provides information about the spectral content of the phase (or time) noise of the signal, as explained in ITU-T G.810, SERIES G [B13].

3.2.2 Time interval error

The time interval error (TIE) is the difference between two time intervals, each delimited by two event occurrences; the first interval is in the actual waveform and the other is in the ideal waveform.

The TIE is expressed as follows in Equation (3):

$$\begin{aligned} J_{TIE,n,L} &= \left(t_{n+L} - t_n\right) - \left(t_{id,n+L} - t_{id,n}\right) \\ &= \left(t_{n+L} - t_{id,n+L}\right) - \left(t_n - t_{id,n}\right) \\ &= J_{TE,n+L} - J_{TE,n}, \end{aligned} \quad (3)$$

where

L+1 is the number of the events that have occurred within the considered time intervals (see DSC AN-1 [B2] and ITU-T G.810, SERIES G [B13]

3.2.2.1 TIE track

TIE is typically measured over a large number of successive reference instants and displayed as a graph, in which the vertical axis shows TIE and the horizontal axis is time (DSC AN-1 [B2]). Plotting a TIE-versus-time curve reveals repeating patterns that indicate a modulation or other periodic components (Tektronix 2012) [B30]. With a continuous TIE track, it is possible to monitor changes in the trend of the accumulated error, $TIE_{\Sigma}[n,L]$, defined as follows in Equation (4):

$$TIE_{\Sigma}[n,L] = \sum_{k=1}^{n} J_{TIE,k,L} \quad (4)$$

In the case of nominally periodic signals, since accumulation (4) is a gradual phenomenon, in the presence of a variation in the signal frequency it is possible to observe an inversion in the slope of (4) versus time plot DSC AN-1 [B2].

3.2.2.2 TIE spectrum

The frequency allocation and magnitude of TIE spectral components can be easily estimated by using well-known frequency-domain transforms, such as the DFT (Tektronix, 2005 [B29]). One of the benefits of spectral analysis of the TIE-versus-time curve is that periodic timing error components, otherwise hidden by wideband noise, are often clearly distinguishable (Tektronix, 2012 [B30]).

3.2.2.3 TIE histogram and statistics

If the number N of TIE measurements is suitably large ($N > 100$), the related histogram may provide a good estimate of the TIE PDF (Tektronix, 2012 [B30]). For example, the identification of the frequency of occurrence of the $J_{TIE,n,L}$ values for all the transitions in a digital signal is a key application of the TIE histogram. The TIE histogram is of particular importance to separate random jitter from deterministic jitter (see3.5 and 3.6).

The maximum time interval error (MTIE) is the maximum value $\left|J_{TIE,n,L}\right|$ within an observation interval whose time indexes are in the range $[n_1, n_2]$. MTIE is estimated by using Equation (5) as followings:

$$MTIE(n_1, n_2, L) = \max_{n_1 \le n \le n_2} \left|J_{TIE,n,L}\right| \quad (5)$$

3.3 Period jitter

Period jitter, *PEJ*, is the jitter in the period of a repetitive signal or its waveform (IEC 60469:2013 [B9]). It depends on two reference instants of the waveform identifying a period and shall be expressed in terms of TIE as follows in Equation (6):

$$PEJ_n = \left(t_{n+1} - t_n\right) - \left(t_{id,n+1} - t_{id,n}\right) = J_{TE,n+1} - J_{TE,n} = J_{TIE,n,1} \quad (6)$$

where

$t_{id,n+1} - t_{id,n} = T_{id}$ is the ideal period

$t_{n+1} - t_n = T_n$ is the *n*th observed period

Note that period jitter shall not be confused with periodic jitter, which is discussed in3.6.2, as shown in Figure 3.

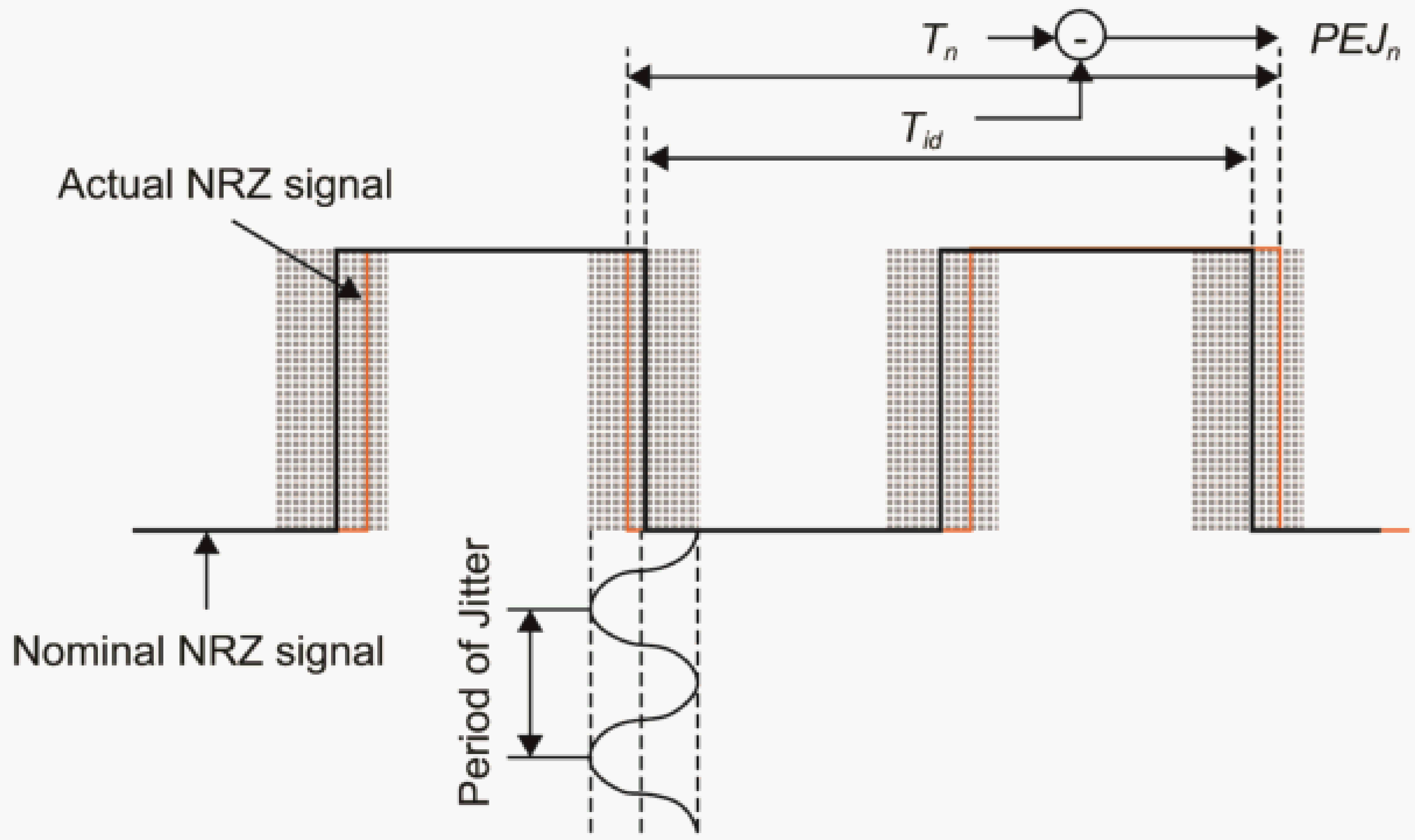

Figure 3—Period and periodic jitter

3.4 Cycle-to-cycle jitter

Cycle-to-cycle jitter (C2C) is defined as the difference between consecutive period durations in an ideally periodic signal. This is a particular case, for two consecutive periods of the waveform, of the cycle-to-nth-cycle jitter defined in (IEC 60469:2013 [B9]). The cycle-to-cycle jitter depends on three events, since two consecutive periods are delimited by two events each, having the mid one in common. It shall be expressed in terms of PEJ as shown in Equation (7):

$$C2C_n = PEJ_{n+1} - PEJ_n . \tag{7}$$

3.5 Random jitter

The random jitter (RJ) is the contribution to the jitter in which successive reference instants cannot be deterministically predicted and, thus, must be described in statistical terms.

RJ results from the superposition of a number of uncorrelated stochastic processes of different kinds (e.g., power supply noise, low-frequency noise [both flicker and generation-recombination], velocity noise, and thermal noise typical of all electronic devices). Nevertheless, the influence is different depending on the function realized by the specific system. For example, in oscillators, RJ could be due to power supply noise and up-converted low-frequency noise. A more detailed study of the sources of RJ by Li [B19] takes into account the semiconductor technology.

By applying the Central Limit Theorem, RJ is modeled as a zero-mean process, with Gaussian probability density function, $f_{RJ}(x)$ (Tektronix, 2012 [B30]):

$$f_{RJ}(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \tag{8}$$

where

x is the random variable contributing to timing error and, consequently, to time interval error $J_{TIE,n,\mathrm{L}}$

It is worth noting that, according to Equation (8), $f_{RJ}(x)$ is completely characterized by its standard deviation σ (or, equivalently, its variance σ^2), which are either analytically calculated or experimentally estimated.

RJ is also assumed to be uncorrelated with other system noise sources. The $f_{RJ}(x)$ is unbounded as the number of observations increases.

In all those cases in which deterministic contribution to the jitter is negligible, the standard deviation can be estimated as reported in5.9.1 of IEC 60469:2013 [B9] starting from a suitable number of observed $J_{TE,n}$ or $J_{TIE,n,\mathrm{L}}$.

Figure 4 shows an example of RJ, both represented as the $J_{TIE,n}$-versus-time curve and summarized by the $J_{TIE,n}$ histogram.

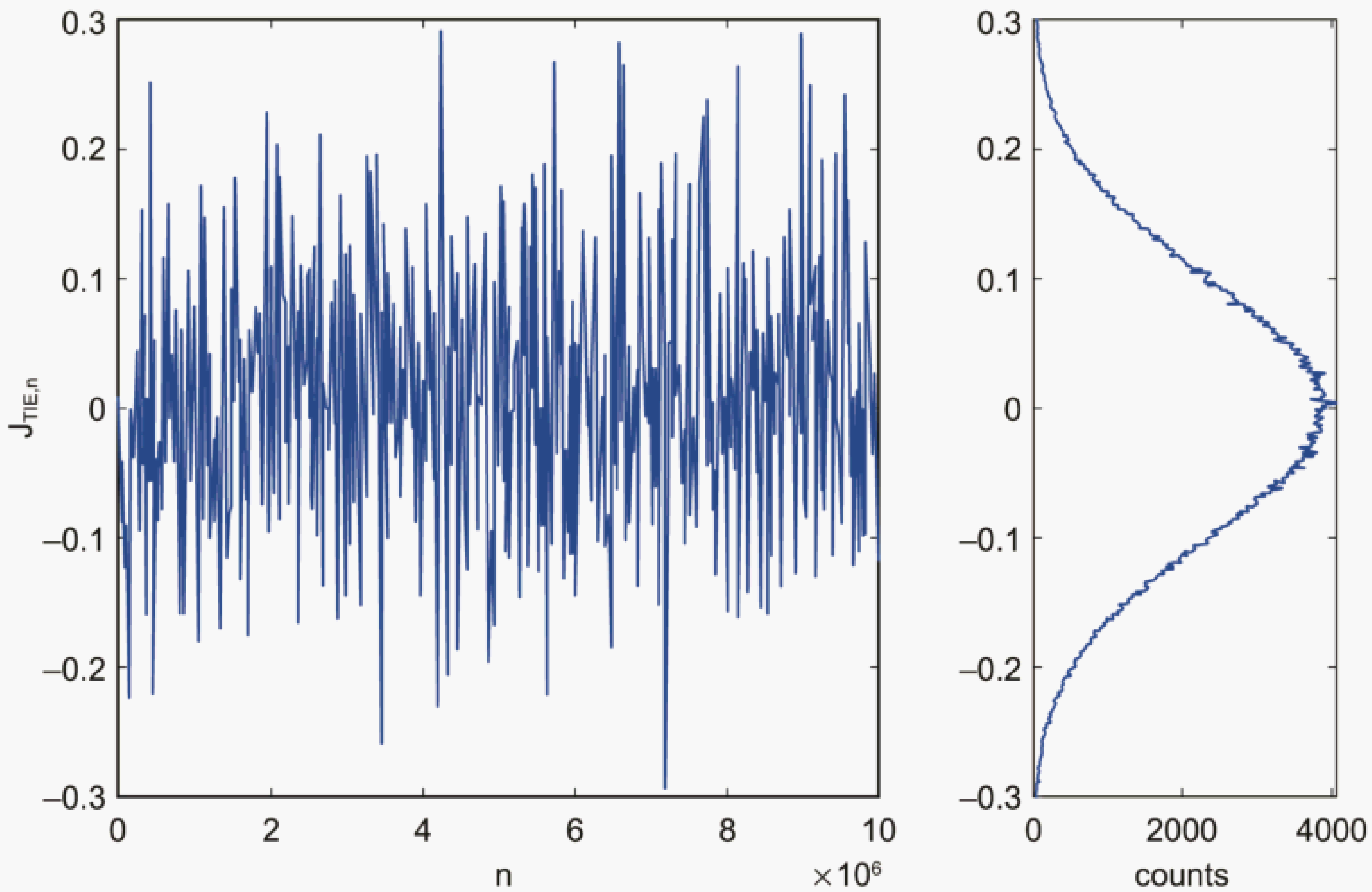

Figure 4—Random jitter represented (left) as $J_{TIE,n}$ -versus-time curve and (right) as a $J_{TIE,n}$ histogram

3.5.1 Wander

Wander is the part of RJ with spectral density below 10 Hz (see IEEE Std 802.3-2008 [B7] and ITU-T G.810, SERIES G [B13]).

In baseband digital signals, wander does not lead to bit errors because the recovered clock easily follows these slow changes in phase. However, depending on its magnitude, to observe wander, observation time intervals on the order of the second are needed.

The most common unit of wander is its magnitude given in second and not in UI, which instead is preferred for jitter measurements above 10 Hz. Moreover, the extremely low frequency components (in the millihertz range) require observation times ranging up to 10^6 s (JDSU [B14]).

3.6 Deterministic jitter

Deterministic jitter (DJ) is the contribution to the jitter in which successive reference instants are deterministically predicted.

DJ has various sources and, in practice, it is often useful to decompose it into its different contributions. DJ exhibits periodic or non-periodic bounded patterns, which will be potentially predicted and reproduced if all the originating sources are properly discovered and analyzed.

DJ is due to various deterministic causes affecting the reference instant at which the waveform feature occurs (Sui, et al. [B27]). Despite its deterministic nature, DJ is regarded as the result of the superposition of more specific and independent contributors, each one modeled with a bounded-domain PDF, as explained in 3.6.4. In particular, as shown in Figure 5, the main contributors to DJ are as follows:

— Data-dependent jitter (DDJ)

— Periodic jitter (PJ)

— Bounded uncorrelated jitter (BUJ)

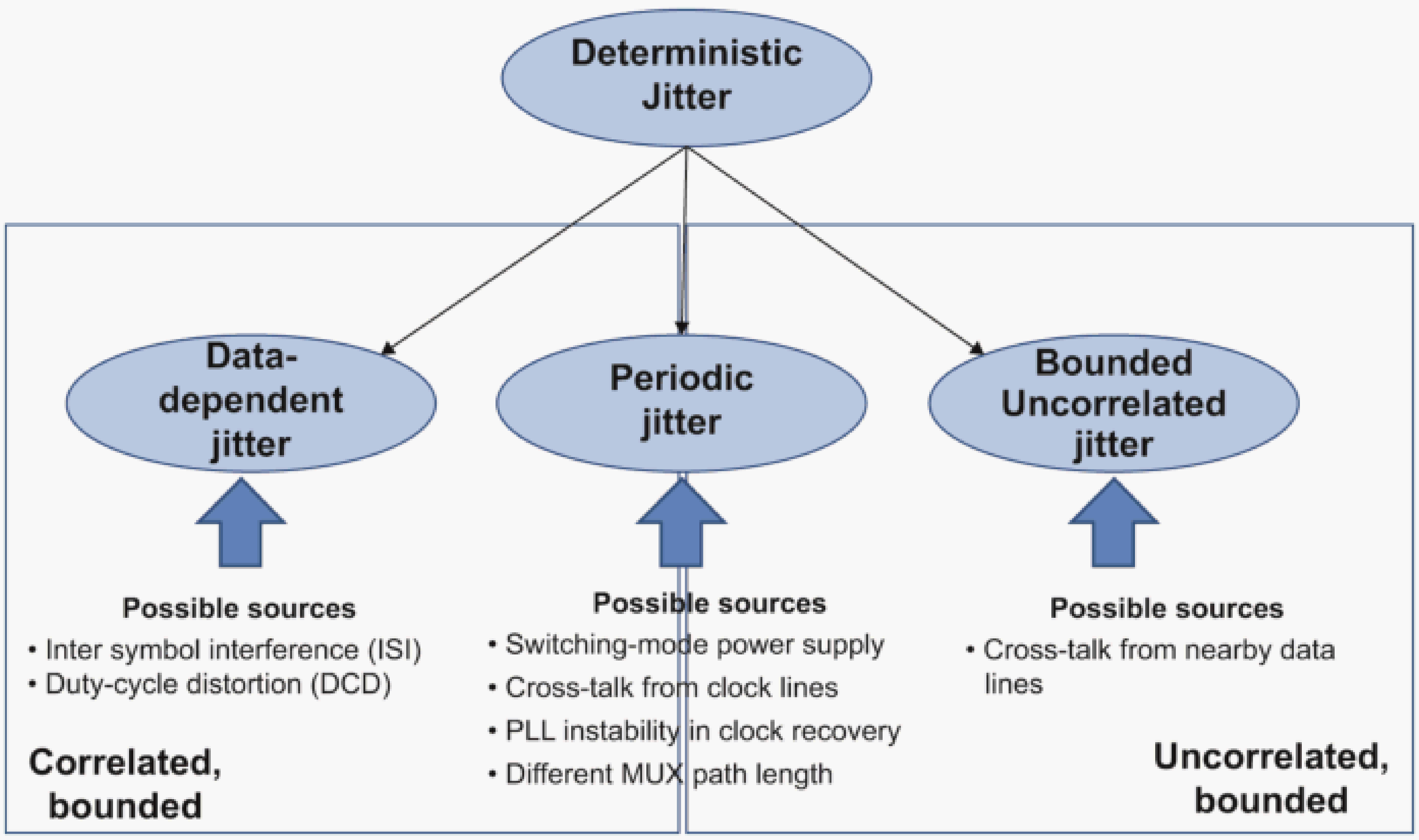

Figure 5—Classification of deterministic jitter contributions and possible sources

In 3.6.1 through 3.6.4, the originating sources and the main models describing the behavior of each DJ contributor are described.

3.6.1 Data-dependent jitter (DDJ)

Data-dependent jitter (DDJ) arises from deviations of the durations of the state level transitions between their ideal values to an actual value that is due to linear or non-linear alteration, modification or distortion of the signal itself that is caused by memory effects.

The effects of DDJ on the signal depend on the signal data pattern. Thus, DDJ is considered as correlated to the signal itself. In high-speed transmission systems, any jitter showing a strong correlation to the signal level being transmitted is also referred to as correlated bounded high probability jitter (CBHPJ) (see IA

OIF-CEI-03.0-2011 [B6], JESD204B.01 [B15], and (Wood, Ferguson, and Evangelista [B32]). For the purposes of this standard, DDJ is assumed to include CBHPJ as well.

3.6.1.1 DDJ sources

DDJ usually originates from the following sources (see Moreira and Werkmann [B21]) and Renesas Electronics Corporation [B24]):

- Inter symbol interference (ISI).

  When ISI is present, amplitude variations in the waveform for a given UI may be introduced by waveform features (such as a slow settling after a waveform transition) and aberrations that have occurred in previous UIs. This will advance or delay the signal transition instants throught a reference level with respect to an ISI-free scenario, thus introducing jitter that depends on the transmitted data pattern (Renesas Electronics Corporation [B24]). For example, transmitting the sequence 00100111 may generate DDJ that is different from that when the sequence 1001001 is transmitted (JESD204B.01 [B15]). ISI has four main causes:

  - Limited analog bandwidth of the communication channel, which changes the profile of the transmitted or received signal depending on the signal's analog bandwidth, with the result that the state levels associated with a given bit are influenced by the preceding bits.
  - Nonlinear phase response of the communication channel causes frequency-dependent group delays, which alters the relative phase of the communication signal's spectrum. This will cause changes in the transistion durations and reference instants for the pulses in the communication signal and is dependent on the transition density of the communication signal.
  - Reflections due to possible impedance mismatch along transmission lines, at transmission line junctions, or at transmission line terminations.
  - Frequency-dependent skin effect and dielectric losses of the transmission lines that extend the transition duration and decrease the amplitude of the propagating pulses in the data signal.
- Duty cycle distortion (DCD). DCD is caused by a time difference between the positive-going and the negative-going transitions of the signal of interest. As a consequence, the time interval between positive-going transitions and subsequent negative-going transitions is different than the time interval between negative-going transitions and subsequent positive-going ones. The difference between these two intervals and the nominal UI gives rise to a two-valued jitter sequence. Additional DCD sources are an inaccurate setting of the threshold level, or an offset affecting the signal amplitude.

While TIE is defined according to a single type of event (e.g., positive-going or negative-going transitions of a signal), DCD explicitly depends on the relationship between two different specific events: the positive-going and negative-going transitions of a two-level signal. Therefore, DCD will be related to TIE if the two transitions are given their own TIE values, respectively. As such, the DCD is given in Equation (9):

$$DCD_n = J_{TIE_2,n,1} - J_{TIE_1,n,1} = PEJ_{2,n} - PEJ_{1,n} \tag{9}$$

3.6.1.2 DDJ models

Under the hypothesis that a stationary system with known input-output characteristic is fed by a known periodic or pseudorandom signal, it is possible to calculate both the values assumed by the DDJ as a function of time and the relative frequencies of occurrence of such values in a given observation window. A well-accepted approach is to treat the relative frequencies of occurrence of DDJ values as probabilities, describing DDJ through a suitable probability density function (see Kim, Kim, and Lombardi [B16], Moreira and Werkmann [B21], and Sun, Li, and Wilstrup [B28]). Such probabilities are often estimated from repeated jitter measurements, eventually using deconvolution techniques to separate DDJ from RJ, under the assumption that DJ and RJ are statistically independent (see Moreira and Werkmann [B21] and Sun, Li, and Wilstrup [B28]). Since DDJ typically assumes a discrete set of values in a bounded domain, its probability density function will be modeled as shown in Equation (10):

$$f_{DDJ}(x) = \sum_{n=1}^{N} P_n \cdot \delta(x - x_n) \tag{10}$$

where

$\delta(\ )$	is the Dirac impulse function
N	is the number of possible reference instant deviations x_n
P_n	is the probability of occurrence of the n-th reference instant deviation x_n
x	is the DDJ contribution to timing error

When only two reference instant deviations (x_1, x_2) are clearly dominant in the observed signals and with ultimately the same probability ($P_1 \cong P_2 \cong 1/2$), then Equation (10) will be reasonably simplified into the so-called dual-Dirac model shown in Equation (11):

$$f_{DDJ}(x) = \frac{1}{2}\delta(x - x_1) + \frac{1}{2}\delta(x - x_2) \tag{11}$$

This dominance of two reference instant deviations occurs, for example, if DCD prevails over ISI contributions and/or if the signal exhibits a clock-like periodic or quasi-periodic pattern. Model (11) is also commonly used for high probability jitter (HPJ) (IA OIF-CEI-03.0-2011 [B6]), which is loosely defined in telecommunications as the "jitter that reaches its peak value when several thousand bits are transferred across the link" (Wood, Ferguson, and Evangelista [B32]). Moreover, Equation (11) is commonly used for jitter estimation (see Renesas Electronics Corporation [B24] and Kim, Kim, and Lombardi [B16]).

3.6.2 Periodic jitter (PJ)

Periodic jitter (PJ) will be either correlated or uncorrelated to the signal of interest, depending on whether the PJ source is caused by the signal itself or not. A distinctive feature of the PJ is the presence of one or multiple identifiable peaks in the spectral density. If the PJ exhibits a periodic pattern at a rate which is an integer or rational submultiple of the signal rate, then it will be referred to as sub-rate jitter (SRJ).

PJ is not to be confused with period jitter, which is defined in 3.3.

3.6.2.1 PJ sources

PJ results from the periodic phase modulation of the signal considered. Sources of such a phase modulation include the following (see Tektronix, 2012 [B30] and Sui, et al. [B27]):

- Cross-talk between transmission lines from periodic signals transmitted over nearby media. If such signals are synchronous and/or in phase with the signal affected by PJ, PJ shall be regarded as correlated. Otherwise, it is uncorrelated to the signal of interest.
- Cross-talk from external electromagnetic sources in the sorrounding environment, either unintentional or intentional (such antennas), directly coupled to the transmission line of interest, or coupled to the power lines.
- Electromagnetic interferences from ac or switching power supply circuitry.
- Instability of the phase-locked loop (PLL) circuit used for clock recovery.
- Different propagation delays along different but coupled processing paths and/or communication links (e.g., due to a multiplexer driven by a periodic signal). This is a typical source of SRJ.

3.6.2.2 PJ models

The periodic modulating signal causing PJ potentially exhibits any shape. Among others, the two most common types of periodic signals originating PJ are as follows:

- Rectangular pulse (e.g., due to clock signals). In this case PJ shall be modeled by a bounded-domain probability density function similar to Equation (11), i.e.,

  $$f_{PJ}(x) = \frac{1}{2}\delta(x - x_L) + \frac{1}{2}\delta(x - x_H) \tag{12}$$

  where

  x_L is the reference instant corresponding to negative-going transition of the rectangular pulse
  x_H is the reference instant corresponding to postive-going transition of the rectangular pulse
- Sinusoidal (e.g., due to power supply waveform). In this case PJ follows a bounded-domain probability density function given by Equation (13):

  $$f_{PJ}(x) = \begin{cases} \dfrac{1}{\pi\sqrt{\left(x_b^{\,2} - x^2\right)}}, & |x| < x_b \\ 0, & |x| \geq x_b \end{cases} \tag{13}$$

  where
  x_b depends on the amplitude of the sinusoidal signal

In the case of complex periodic modulating signals that are given by the superposition of multiple independent contributions, the PDF of PJ results from the convolution of the PDFs, Equation (12) to Equation (13), associated with each individual contribution (see Moreira and Werkmann [B21]).

3.6.3 Bounded uncorrelated jitter (BUJ)

The bounded uncorrelated jitter (BUJ) is a non-periodic contribution to deterministic jitter in a two-state (binary) or multilevel signal that is associated with the signal transitions but which shows no correlation to the signal itself. In high speed transmission systems, this definition is compatible with the so-called uncorrelated bounded high probability jitter (UBHPJ) (see IA OIF-CEI-03.0-2011 [B6], JESD204B.01 [B15], and Wood, Ferguson, and Evangelista [B32]). If BUJ is not properly recognized and estimated, its contribution leads to an overestimation of RJ (see ITU-R Recommendation BT.1363 [B12]).

3.6.3.1 BUJ sources

The BUJ is caused by all phenomena recognized as deterministic and non-periodic, but that are not correlated to the signal considered (IEEE Std 802.2-2008 [B7]). BUJ usually originates from the following sources:

— Cross-talk between adjacent data-carrying lines, one called the aggressor and the other the victim, where the aggressor signals are neither periodic (e.g., clock) nor correlated to the signal considered (victim) (see Kuo, et al. [B18]). BUJ is bounded because of finite electromagnetic coupling between nearby lines.
— Bounded power supply noise injected at the transmitting and receiving ends of the transmission line and clipped by active circuits (see ANSI Technical Report Working Draft T11.2 [B1]).

3.6.3.2 BUJ models

Although no unique BUJ model currently exists, experimental evidence shows that a zero-mean bounded Gaussian probability density function can describe the joint effect of multiple BUJ sources reasonably well (see Kuo, et al. [B18] and Shimanouchi, Li, and Chow [B25]). Such a probability density function is defined in Equation (14) as follows:

$$f_{BUJ}(x)=\begin{cases}\dfrac{C_{BUJ}}{\sqrt{2\pi}\sigma_{BUJ}}e^{-\frac{x^2}{2\pi\sigma_{BUJ}^2}} & |x|\le BUJ_{pk}\\ 0 & |x|>BUJ_{pk}\end{cases} \tag{14}$$

where

BUJ_{pk} is the peak jitter value
σ_{BUJ} is the standard deviation of the distribution considered

$$C_{BUJ} = \frac{1}{\Phi\left(\frac{BUJ_{pk}}{\sigma_{BUJ}}\right) - \Phi\left(-\frac{BUJ_{pk}}{\sigma_{BUJ}}\right)}$$

is a normalization coefficient (with $\Phi(\cdot)$ being the cumulative distribution function of a zero-mean unit-variance normal random variable), which makes the integral of (14) in the interval $\left[-BUJ_{pk}, BUJ_{pk}\right]$ equal to 1.

3.6.4 DJ model

DJ is assumed to be the superposition of its main independent contributions, i.e., DDJ, PJ, and BUJ. Therefore, the probability density function, f_{DJ}, of DJ is given by the linear convolution of the individual models described in 3.6.1.2, 3.6.2.2, and 3.6.3.2, i.e., as shown in Equation (15):

$$f_{DJ}(x) = f_{DDJ}(x) * f_{PJ}(x) * f_{BUJ}(x) \tag{15}$$

where

$*$ denotes the linear convolution operator

3.7 Total jitter

Total jitter (TJ) is the sum of the main independent contributions of jitter, i.e., the RJ and DJ. Therefore, the probability density function f_{TJ} of the TJ is given by the linear convolution of the models described in Equation (8) and Equation (15) (see Hancock [B5]):

$$f_{TJ}(x) = f_{RJ}(x) * f_{DJ}(x) \tag{16}$$

3.8 RMS and peak-to-peak jitter

The RMS total jitter, TJ_{rms}, is the root-mean-square value of the deviations of interest (see Maxim Integrated [B20]). More precisely, these deviations are either the TE, as defined in 3.2.1, or the TIE, as defined in 3.2.2. In either case, the value of the jitter must be provided together with the indication of whether the timing error or the time interval error has been considered.

In terms of the jitter probability density function, $f_{TJ}(x)$, the definition of TJ_{rms} is given by Equation (17):

$$TJ_{rms} = \sqrt{\int_{-\infty}^{+\infty} (x-\mu_J)^2 f_{TJ}(x) dx} \tag{17}$$

where

f_{TJ}	is either the PDF of TE or the PDF of TIE
$\mu_J = \int_{-\infty}^{+\infty} x \cdot f_{TJ}(x) dx$	is the associated possible non-zero mean

The same quantity can be estimated using a time average computed on the sequence of deviation values:

$$TJ_{rms} = \sqrt{\frac{1}{N-1}\sum_{n=1}^{N}\left(J_n - \bar{J}\right)^2} \tag{18}$$

where

J_n is either $J_{TE,n}$ or $J_{TIE,n,L}$

$\bar{J} = \frac{1}{N}\sum_{n=1}^{N} J_n$

The TJ_{rms} is expressed in units that are scale appropriate or UI. In the second case, the symbol UI_{rms} shall be used (see Redd [B23]).

The peak-to-peak jitter, TJ_{pp}, is the difference between the maximum and the minimum deviations (see Frenzel [B4]), which will be either $J_{TE,n}$ or $J_{TIE,n,L}$. In either case, the value of the jitter must be provided together with the indication of whether TE or TIE is considered.

If the deviation has an unbounded distribution then, strictly speaking, TJ_{pp} is not defined (see Kundert [B17]). In that case, TJ_{pp} shall be defined as the amplitude of a probability interval, i.e., of the interval of deviations covering the $f_{TJ}(x)$ PDF with a specified probability p (Figure 6). For a symmetric zero-mean distribution of the deviations the probability interval is symmetric around zero and $TJ_{max}>0$, $TJ_{min}=-TJ_{max}$, then $TJ_{max}=TJ_{PP}/2$ and:

$$\text{Prob}\left(|TJ| \le \frac{TJ_{pp}}{2}\right) = p \tag{19}$$

Units for peak-to-peak jitter measurements are given in scale appropriate units or UI. In the second case, the symbol UI_{pp} shall be used (see Redd [B23]).

In the case of a communication equipment, the probability p is directly related to the target bit-error rate (BER) of the equipment (see 3.9) (see Kundert [B17]).

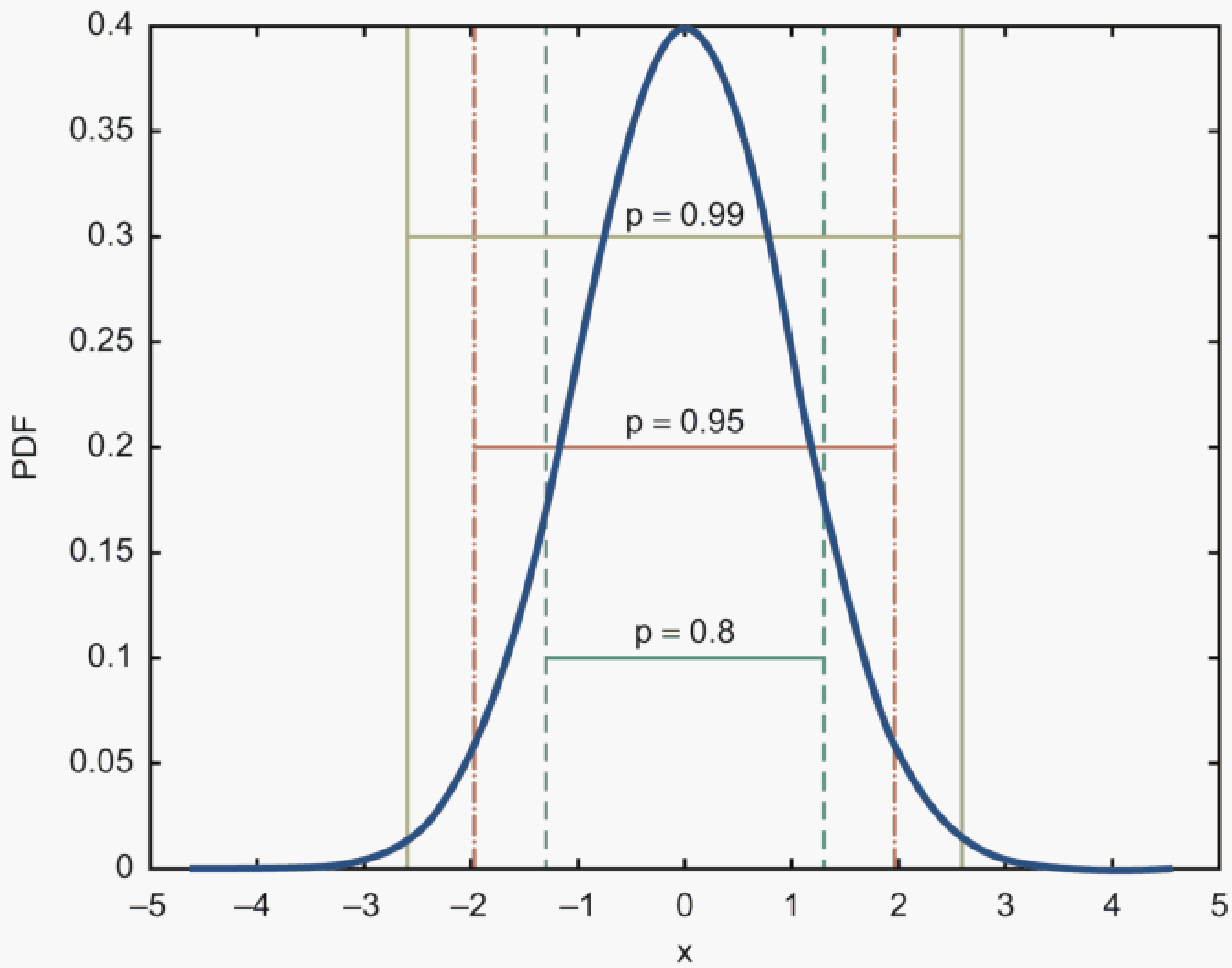

Figure 6—Example of peak-to-peak jitter for Gaussian PDF $f_{TJ}(x)$ with different probabilities

3.9 Bit-error rate (*BER*)

In digital communication equipment, when the total jitter exceeds a certain magnitude, a bit error is generated.

On the basis of the above considerations, for a symmetric zero-mean distribution of the deviations it can be demonstrated that the BER is given by Equation (20):

$$\text{BER}=\text{D}\cdot\text{Prob}\left(|TJ|>\frac{TJ_{pp}}{2}\right) \tag{20}$$

where D is the *transition density* in the digital waveform encoding the bit sequence, i.e. the probability that two consecutive bits are different. It is called transition density because, when in a digital waveform two consecutive bits are different, a waveform transition occurs between the associated time intervals. The usually assumed value for the transition density is D=0.5, associated to a truly random binary sequence. It is also frequently considered the value D=1, associated to a binary signal following a clock pattern. In the latter case, indeed, there is also a transition between two consecutive bit periods.

In the common case where only a random Gaussian jitter is present, TJ_{pp} is given by Equation (21):

$$TJ_{pp}=k\cdot TJ_{rms} \tag{21}$$

where the coefficient k is linked to the *BER* by the relation:

$$BER = 1 - \Phi\left(\frac{k}{2}\right) \tag{22}$$

and $\Phi(z)$ is the normal standard cumulative function. Expressing k as a function of BER, the relation is

$$k = 2 \cdot \Phi^{-1}\left(1 - BER\right) \tag{23}$$

where $\Phi^{-1}(\alpha)$ is the inverse cumulative, i.e., the standard normal quantile function. By applying Equation (22) and Equation (23), Table 1 and Table 2 are obtained.

Table 1—Values of *BER* corresponding to some integer values of the coefficient *k* for a Gaussian distribution of the jitter

BER	k (D = 0.5)
$1.349 \cdot 10^{-3}$	6
$2.326 \cdot 10^{-4}$	7
$3.167 \cdot 10^{-5}$	8
$3.398 \cdot 10^{-6}$	9
$2.867 \cdot 10^{-7}$	10
$1.899 \cdot 10^{-8}$	11
$9.866 \cdot 10^{-10}$	12
$4.016 \cdot 10^{-11}$	13
$1.280 \cdot 10^{-12}$	14
$3.191 \cdot 10^{-14}$	15
$6.221 \cdot 10^{-16}$	16

Table 1 is valid for D=0.5. For D=1, the values of BER are simply doubled.

Table 2—Values of *k* corresponding to some power of ten *BER* values for a Gaussian distribution of the jitter

BER	k (D = 0.5)	k (D = 1)
10^{-3}	6.1805	6.5811
10^{-4}	7.4380	7.7812
10^{-5}	8.5298	8.8343
10^{-6}	9.5068	9.7833
10^{-7}	10.399	10.653
10^{-8}	11.224	11.461
10^{-9}	11.996	12.219
10^{-10}	12.793	12.934
10^{-11}	13.412	13.613
10^{-12}	14.069	14.261
10^{-13}	14.698	14.882
10^{-14}	15.301	15.479
10^{-15}	15.883	16.054
10^{-16}	16.444	16.610

3.10 Phase noise

The phase noise is the random fluctuations in the phase of a periodic signal. It is a characterization in the frequency domain of the timing error in a periodic signal, for example, a carrier sinusoid in a transmission system.

Given the ideal sinusoidal signal, $v_{id}(t) = A_0 \sin(2\pi f_0 t)$, the non-ideal signal, $v(t)$, is modeled by Equation (24):

$$v(t) = [A_0 + \varepsilon(t)]\sin(2\pi f_0 t + \varphi(t)) \tag{24}$$

where

A_0	is the nominal peak amplitude
$\varepsilon(t)$	is the deviation from the nominal amplitude
f_0	is the nominal frequency
$\varphi(t)$	is the random phase deviation

The signal in Equation (24) is affected by random phase deviations and by amplitude instability (see Figure 7, which is defined in terms of instantaneous, normalized amplitude deviation:

$$a(t) \equiv \varepsilon(t) / A_0 \tag{25}$$

The amplitude instability is a separate occurrence that is not covered in this standard. We will assume $a(t)$ = 0 in the following discussion.

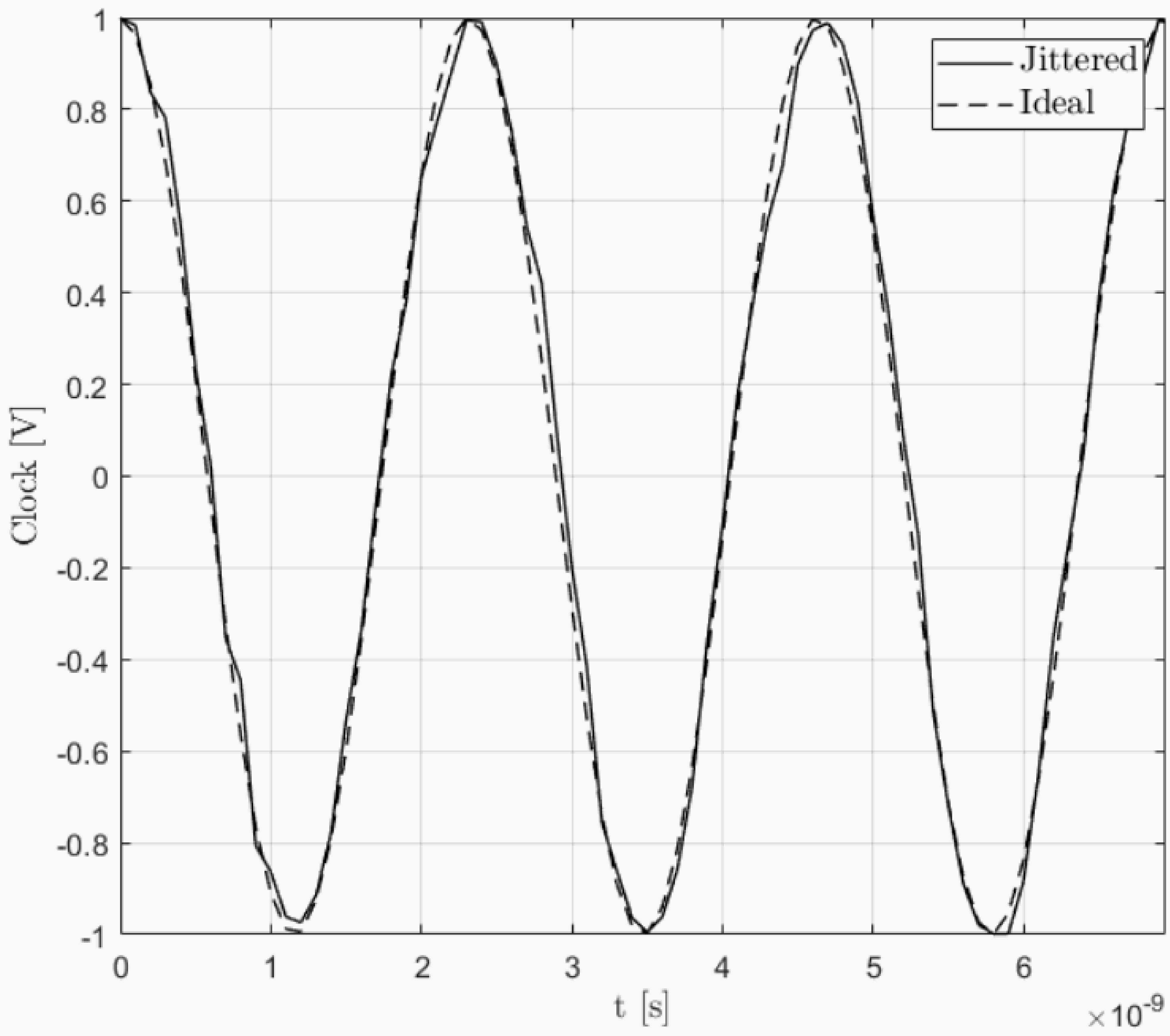

Figure 7—Time-domain plot of a sinusoidal signal affected by phase noise and its ideal counterpart. The jittered signal is not affected by amplitude instability.

The phase noise is quantified as the single-sideband-phase-noise-to-carrier ratio, $\mathcal{L}(f)$ (see IEEE Std 802.3-2008 [B7]):

$$\mathcal{L}(f) = \frac{S_\varphi(f)}{2} \tag{26}$$

where $S_\varphi(f)$ is the one-sided PSD of $\varphi(t)$. $\mathcal{L}(f)$ is measured in squared radians per hertz (rad^2/Hz).

$\mathcal{L}(f)$ is easy to measure in practice under the following conditions (see Renesas Electronics Corporation [B24]):

a) $\varepsilon(t)$ must be negligible

b) the magnitude of $\varphi(t)$ must be conveniently small, i.e., its rms value must be not greater than 0.01 rad (see IEEE Std 802.3-2008 [B7]), and its peak value must be not exceed 0.2 rad (see Renesas Electronics Corporation [B24]). This is the "small angle" hypothesis.

Under these conditions, by following the demonstration reported in Annex A:

$$\mathcal{L}(f)=\frac{S_\varphi(f)}{2}\simeq\frac{S_v(f+f_0)}{A_0^2/2}=\frac{P_{SSB}(f)}{P_v} \tag{27}$$

where

S_v is the one sided PSD of *v(t)*,

P_{SSB} is the S_v evaluated at the offset frequency f from the carrier frequency f_0 $P_{SSB}(f)=S_v(f+f_0)$

$P_v=A_0^2/2$ is the carrier power

If condition a) and condition b) do not hold, Equation (27) is still valid, but it is no longer possible to compute $\mathcal{L}(f)$ using Equation (27). In such cases it is necessary to resort to sophisticated measurement techniques, equivalent to a phase demodulation of the signal shown in Equation (24). Such techniques are out of the scope of this standard.

The RMS phase noise over a frequency range $[f_a, f_b]$ is the square root of the following integral, where f_a and f_b is chosen as in (see IEEE Std 802.3-2008 [B7]):

$$\varphi_{rms}=\sqrt{\int_{f_a}^{f_b}S_\varphi(f)df}=\sqrt{2\int_{f_a}^{f_b}\mathcal{L}(f)df} \tag{28}$$

See examples in Figure 8 and Figure 9.

The RMS phase jitter is shown in Equation (29):

$$J_{\varphi,rms}=\frac{\varphi_{rms}}{2\pi f_0} \tag{29}$$

which is expressed in time units (appropriately scaled units or UI).

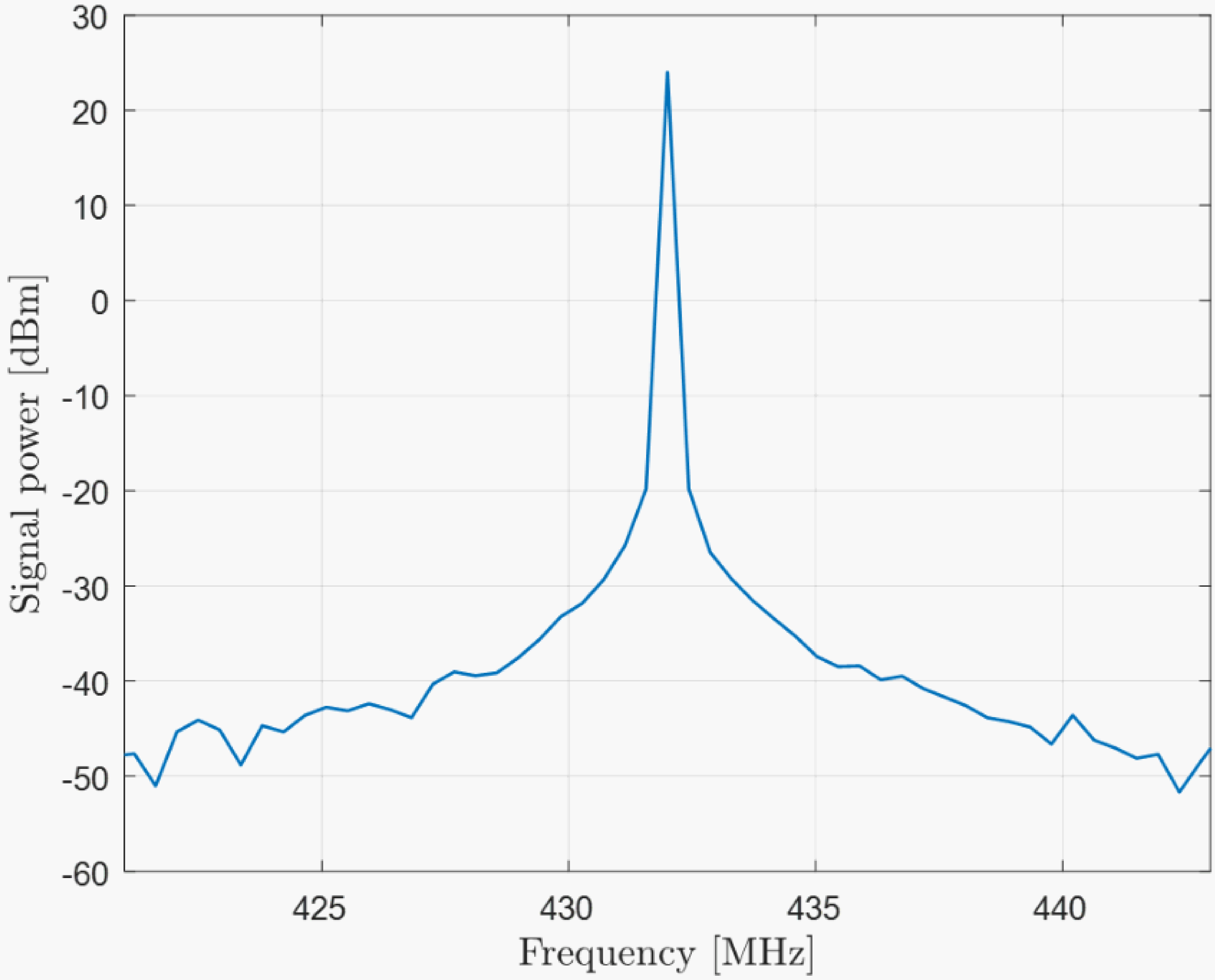

Figure 8—Example of power spectral density $S_v(f)$ of a jitter-affected sinusoidal carrier $v(t)$. The carrier frequency is f_0 = 432MHz. The effect of the timing jitter in the sampling instants is a phase modulation.

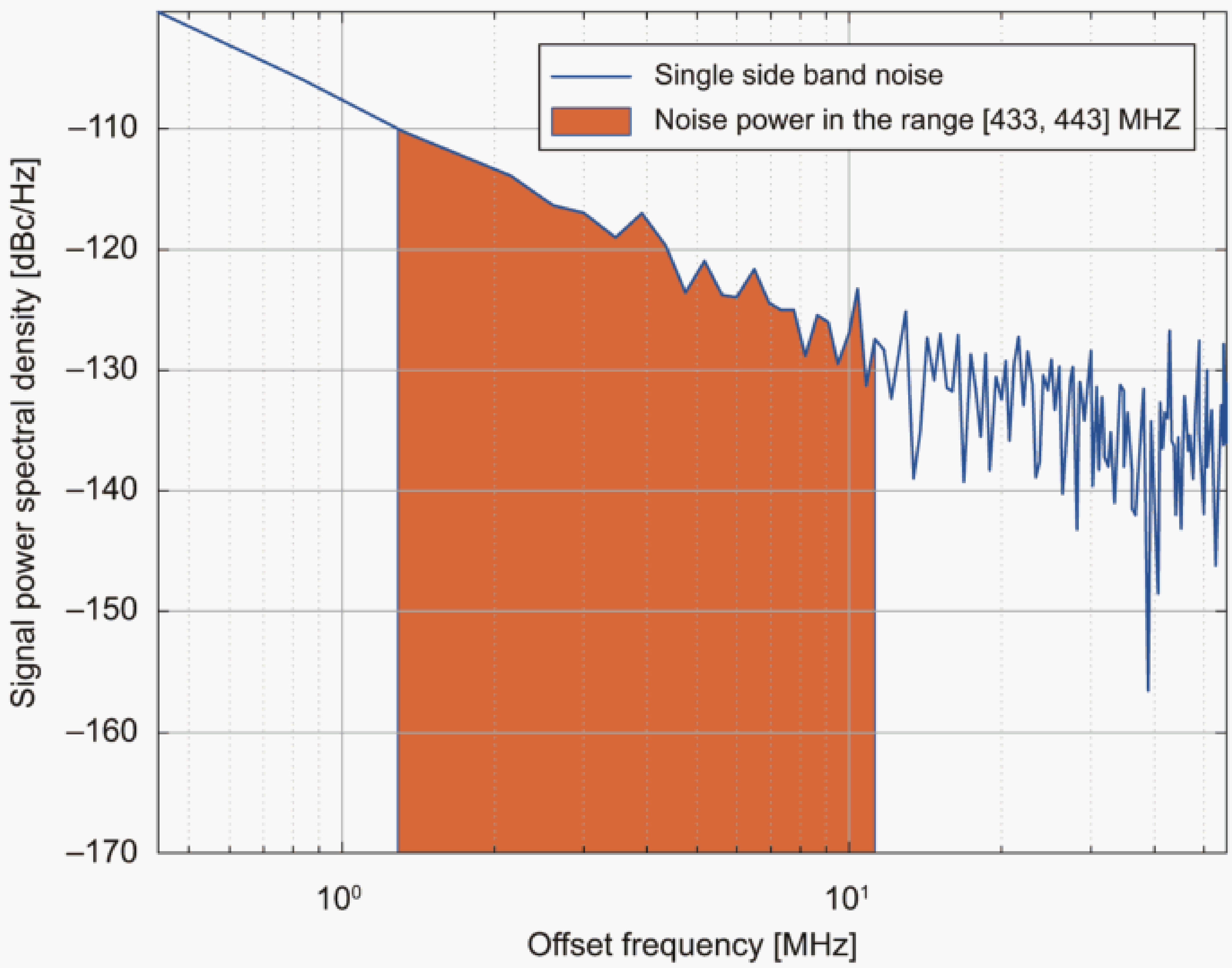

Figure 9— Evaluation of $\mathcal{L}(f)$ using eq.(27), i.e. in terms of single sideband spectral noise, expressed in decibel-under carrier units, evaluated at offset frequencies (blue line). The integral (28) is also highlighted.

Annex A

(informative)

Derivation of $\mathcal{L}(f)$

A.1 General

The ideal signal, $v_{id}(t)$ and the actual signal affected by phase noise, $v(t)$ are:

$$v_{id}(t) = A_0 sin(2\pi f_0 t) \tag{A.1}$$

$$v(t) = \left[A_0 + \varepsilon(t)\right]\sin\left(2\pi f_0 t + \varphi(t)\right) \tag{A.2}$$

The required conditions are:

a) The amplitude instability ε(t) must be negligible.

b) The peak value of φ(t) must be conveniently small (“small angle” hypothesis, see 3.10 for details).

The two-sided power spectral density, of the jitter signal is:

$$\breve{S}_J(f) = PSD\left[J(t)\right] \tag{A.3}$$

and the two-sided power spectral density of the phase noise signal is

$$\breve{S}_\varphi(f) = PSD\left[\varphi(t)\right] \tag{A.4}$$

Due to the time-domain relation $J(t) = \varphi(t)/(2\pi f_0)$, the relation between the two PSDs is:

$$\breve{S}_\varphi(f) = (2\pi f_0)^2 \breve{S}_J(f) \tag{A.5}$$

Under the condition b) in 4.10, the following approximation of $v_{id}(t)$, based on the Taylor expansion, is acceptable:

$$\begin{aligned} v(t) &= v_{id}\left(t + J(t)\right) \\ &\cong v_{id}(t) + \dot{v}_{id}(t)\cdot J(t) \end{aligned} \tag{A.6}$$

where $\dot{v}_{id}(t)$ denotes the first derivative with respect to *t*. The ideal signal $v_{id}(t)$, therefore, is seen as affected by an additive amplitude noise $e(t)$ given by:

$$e(t) = v(t) - v_{id}(t)$$

which after substitution of Equation (A.6) for $v(t)$ and with condition a), is:

$$e(t) \cong A_0 (2\pi f_0) \cos(2\pi f_0 t) \cdot J(t) \tag{A.7}$$

On the basis of Equation (A.7) the PSD, $\breve{S}_e(f)$ of the additive noise $e(t)$ is the PSD $\breve{S}_J(f)$, shifted at the frequencies f_0 and $-f_0$, and multiplied by the proper factor (one fourth of the power of the sinusoid):

$$\breve{S}_e(f) \cong \frac{A_0^2}{2}(2\pi f_0)^2 \left[\frac{1}{4}\breve{S}_J(f-f_0) + \frac{1}{4}\breve{S}_J(f+f_0)\right] \tag{A.8}$$

that is

$$\breve{S}_e(f) \cong \frac{1}{4}\frac{A_0^2}{2}\left[\breve{S}_\varphi(f-f_0) + \breve{S}_\varphi(f+f_0)\right] \tag{A.9}$$

The one-sided PSD of $e(t)$ is

$$S_e(f) \cong \frac{1}{2}\frac{A_0^2}{2} S_\varphi(f-f_0) \tag{A.10}$$

and the one-sided PSD of $v(t)$, $S_v(f)$, is obtained by adding a spectral line of power $A_0^2/2$ (the power of the carrier) at the frequency f_0, i.e.

$$S_v(f) \cong \frac{A_0^2}{2}\left[\delta(f-f_0) + \frac{1}{2} S_\varphi(f-f_0)\right] \tag{A.11}$$

Now, by considering the shifted spectrum $P_{SSB}(f) \cong S_v(f+f_0)$, divided by the carrier power $P_v = A_0^2/2$, we obtain:

$$\frac{P_{SSB}(f)}{P_v} = \frac{S_v(f+f_0)}{A_0^2/2} \cong \delta(f) + \frac{S_\varphi(f)}{2} \tag{A.12}$$

Summing up, neglecting the spectral line at the offset frequency $f=0$, the result is

$$\frac{P_{SSB}(f)}{P_v} \cong \frac{S_\varphi(f)}{2} = \mathcal{L}(f) \tag{A.13}$$

A.2 Mathematical model to compute the BER and derivation of Tables 1 and 2

The formulae presented below describe a general mathematical modeling of BER due to Gaussian jitter. It has reformulates and explains (see Moreira and Werkmann [B21]), to improve clarity and to make the discussion closer to the Standard and its symbols.

In order to link the jitter to the BER, we start considering the waveform in Figure A.1.

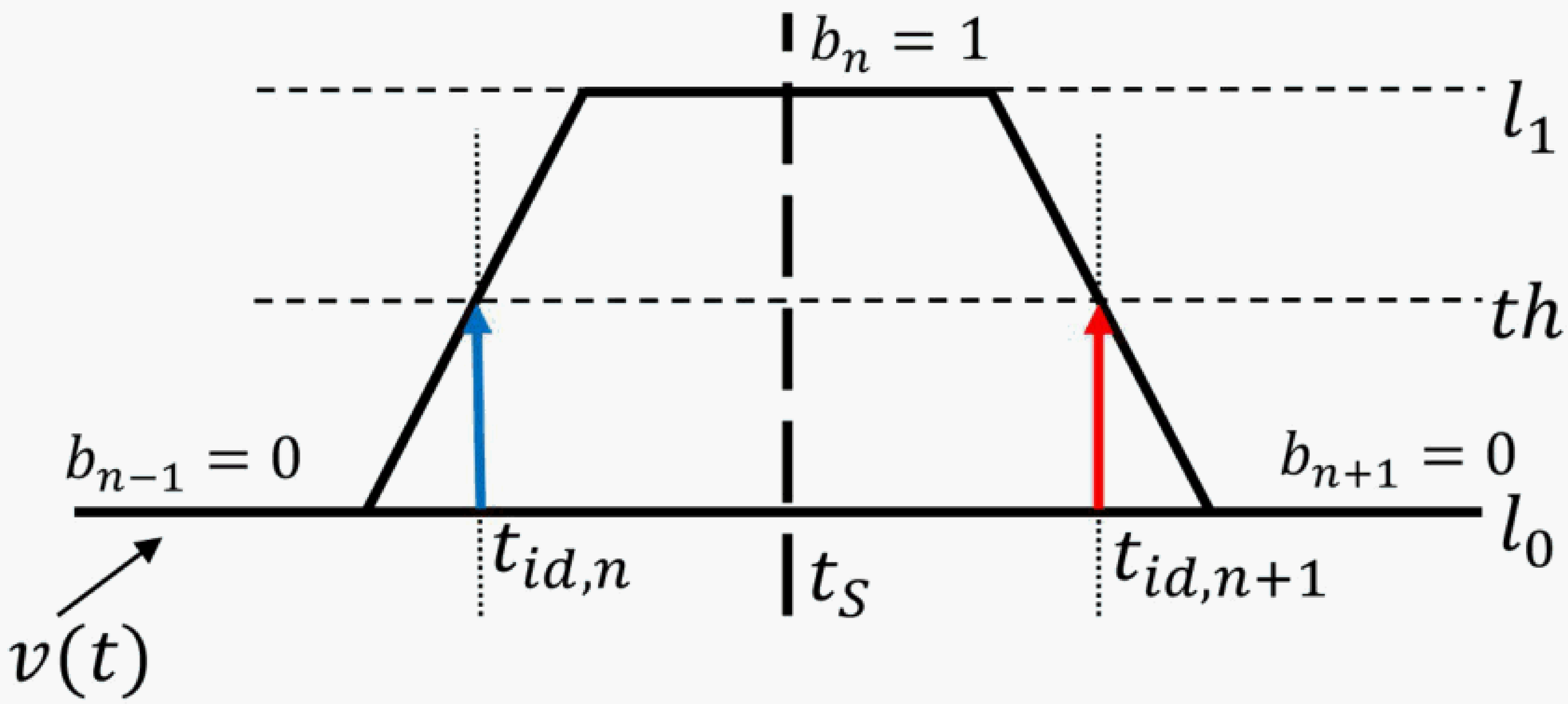

Figure A.1—Example of digital signal $v(t)$ encoding the bit sequence $[b_{n-1}, b_n, b_{n+1}] = [0,1,0]$

The waveform $v(t)$ in the figure is an example of digital signal in a serial interface. It encodes the values of three adjacent bits $[b_{n-1}, b_n, b_{n+1}] = [0,1,0]$. The digital levels of the waveform are l_o and l_1, the threshold level used to measure the value of a bit is $th = (l_0 + l_1)/2$. The transition instants are those of threshold crossing: $v(t) = th$. Their ideal positions are $t_{id,n}$, $t_{id,n+1}$. The time interval $t_{id,n+1} - t_{id,n} = T_{bit}$ does not depend on n and is the period of a bit.

In order to determine the value of b_n, $v(t)$ is measured in the "strobe" instant $t_S = (t_{id,n} + t_{id,n+1}/2)$. The measured value of b_n is $\hat{b}_n$, where:

$$\hat{b}_n = \begin{cases} 1, & v(t_S) \geq th \\ 0, & v(t_S) < th \end{cases} \quad \text{(A.14)}$$

(It must be noted that the values 1 and 0 could be reversed, without any substantial change in the discussion.) Of course, in the ideal situation depicted in Figure A.1, a bit error is not possible.

We consider now the case when actual transition instants t_n, t_{n+1}, different from the ideal ones, due to jitter. They are affected by timing errors $J_{TE,n} = t_n - t_{id,n}$, $J_{TE,n+1} = t_n - t_{id,n+1}$. Obviously, when timing errors are large, a bit error can occur. Figure A.2 represents this situation.

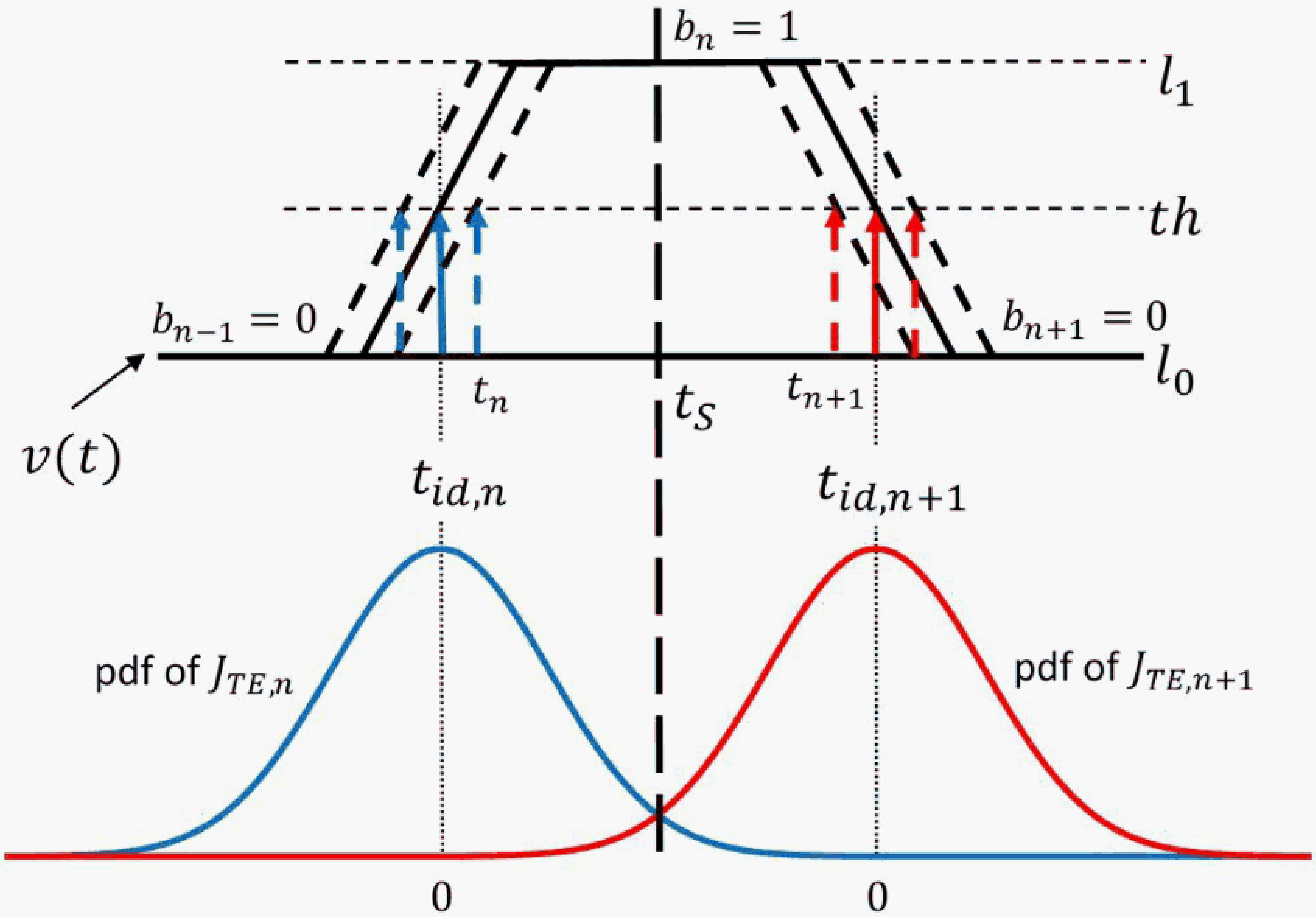

Figure A.2—Digital signal $v(t)$ with transition instants t_n, t_{n+1} affected by timing errors $J_{TE,n}, J_{TE,n+1}$ and Gaussian pdfs of the errors

At the top of the figure, possible transitions of $v(t)$ affected by timing errors are represented with dashed lines. At the bottom of the figure, the pdfs of the timing errors $J_{TE,n}, J_{TE,n+1}$ are represented, centered on the ideal position of the transition times $t_{id,n}$, $t_{id,n+1}$. We consider the special case when $J_{TE,n}, J_{TE,n+1}$, are independent, normally distributed, with mean 0 and standard deviation σ. They can be represented, therefore, by the same random variable $J_{TE} \sim N\left(0, \sigma^2\right)$. The standard deviation is the rms jitter: $\sigma = J_{rms}$.

A bit error occurs when $\hat{b}_n \neq b_n$, that is when either one or the other of the two conditions below occur:

a) $t_n > t_S$ AND $b_{n-1} \neq b_n$

b) $t_{n+1} < t_S$ AND $b_{n+1} \neq b_n$

Annex B

(informative)

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