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Introduction

This standard serves as a standardized method to determine the uncertainty associated with various aspects of petroleum measurement. This standard supersedes API *MPMS* Chapter 13.1, *Statistical Concepts and Procedures in Measurement*, which has been withdrawn. Content from the superseded standard has been merged with this document to prevent duplication and promote alignment of content among the Chapter 13 standards.

This method is based on the 2008 edition of the International Organization of Standards (ISO) *Guide to the Expression of Uncertainty in Measurement* (GUM)-JCGM 100:2008-which was developed to be a guide for the writers of technical standards.

Although this document could be used for analysis of an entire system or facility, that use is outside the scope of the document.

The uncertainty estimate is only as good as the underlying data and engineering judgment. All of the numerical values used and assumptions made must be documented. The statement in Section 3.4 of the ISO *GUM* reproduced below applies to this standard:

“Although this guide provides a framework for assessing uncertainty, it cannot substitute for critical thinking, intellectual honesty and professional skill. The evaluation of uncertainty is neither a routine task nor a purely mathematical one; it depends on detailed knowledge of the nature of the measurand and of the measurement. The quality and utility of the uncertainty quoted for the result of a measurement therefore ultimately depend on the understanding, critical analysis and integrity of those who contribute to the assignment of its value.”

Measurement Uncertainty

1 Scope

This standard reviews basic statistical concepts and uses such to establish a methodology to develop uncertainty analyses for use in writing API *Manual of Petroleum Measurement Standards (MPMS)* documents that are consistent with the ISO *GUM* and NIST Technical Note 1297.

2 Normative References

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

API *Manual of Petroleum Measurement Standards (MPMS)* Chapter 12, *Calculation of Petroleum Quantities, Section 2—Calculation of Liquid Petroleum Quantities Measured by Turbine or Displacement Meters*, 1981

API *MPMS* Chapter 13, *Statistical Aspects of Measuring and Sampling, Section 2—Methods of Evaluating Meter Proving Data*, 1994, Reaffirmed 2011

API *MPMS* Chapter 14.3, (AGA Report No. 3, Part 1), *Concentric, Square-Edged Orifice Meters, Part 1—General Equations and Uncertainty Guidelines*, 2012

API *MPMS* Chapter 22.1, *Testing Protocols—General Guidelines for Developing Testing Protocols for Devices Used in the Measurement of Hydrocarbon Fluids*, 2006, Reaffirmed 2011

Coleman, H.W., and Steele, W.G., *Experimentation and Uncertainty Analysis for Engineers*, Third Edition, New York: John Wiley and Sons, 2009

Harter, H.L., "Tables of Range and Studentized Range," *Annals of Mathematical Statistics*, vol. 31, Beachwood, OH, 1960, pp. 1122–1147

JCGM 100:2008, *Evaluation of measurement data—Guide to the expression of uncertainty in measurement, GUM 1995 with minor corrections*, 2008

JCGM 101:2008, *Evaluation of measurement data—Supplement 1 to the "Guide to the expression of uncertainty in measurement"—Propagation of distributions using a Monte Carlo method, GUM 1995 with minor corrections*, 2008

JCGM 200:2012, *International vocabulary of metrology—Basic and general concepts and associated terms (VIM)*, 2012

Olkin, I., "Range Restrictions for Product-Moment Correlation Matrices," *Psychometrika*, December 1981, vol. 46, Issue 4, pp. 469–472

Taylor, B.N., and Kuyatt, C.E., *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, NIST Technical Note 1297, 1994

Wheeler, D.J., *EMP III: Evaluating the Measurement Process and Using Imperfect Data*, SPC Press, 2006

3 Terms, Definitions, Acronyms, Abbreviations, and Symbols

3.1 Terms and Definitions

For the purposes of this document, the following terms and definitions apply. Terms of more general use may be found in the API *MPMS* Chapter 1, Online Terms and Definitions Database.

The intent is to use terms from JCGM 200:2012 (VIM) when they are available. An important point is that VIM often includes notes that more fully explain the term being defined. These notes are not shown in these definitions. The analyst is encouraged to consult with VIM to obtain a more complete understanding of the terms.

3.1.1

accuracy

See **measurement accuracy**.

3.1.2

bias

See **measurement bias**.

3.1.3

calibration

A set of operations which establish, under specified conditions, the relationship between the values indicated by a measuring device and the corresponding known values indicated when using a suitable measuring standard.

3.1.4

confidence interval

confidence range

range of uncertainty

The range within which the true value is expected to lie with a stated confidence level. See 4.7 for more detail.

3.1.5

confidence level

The degree of confidence that may be placed on an estimated confidence interval. See 4.7 for more detail.

3.1.6

dead band (JCGM 200:2012, 4.17)

Maximum interval through which a value of a quantity being measured can be changed in both directions without producing a detectable change in the corresponding indication. See related term: **hysteresis**.

3.1.7

degrees of freedom (JCGM 100:2008, C.2.31)

dof

The number of terms in a sum minus the number of constraints on the terms of the sum.

3.1.8

direct measurement (adapted from JCGM 200:2008, 2.5)

Comparison of the measurand to a standard where the desired measurement is read from the standard.

EXAMPLE Using a tape measure to measure the length of a measurand.

3.1.9**drift**

See **instrumental drift**.

3.1.10**elemental error source** (adapted from Coleman and Steele)

The most basic component within the hierarchical structure of uncertainty contributors.

3.1.11**error**

See **measurement error**.

3.1.12**given uncertainty** (adapted from JCGM 200:2008, F.2.3)

The numerical uncertainty and confidence interval associated with an elemental error source that has not been estimated in the course of performing the measurement under analysis.

EXAMPLE One of the uncertainty specifications of a device provided by the manufacturer may be, "0.2 % of reading with a confidence level of 90 %." If this information is used in an uncertainty analysis, then it has been "given" to the analyst rather than being experimentally determined.

3.1.13**gross error**

Error in procedure or an error in the execution of procedure.

3.1.14**GUM**

Acronym for JCGM 100:2008, *Evaluation of measurement data—Guide to the expression of uncertainty in measurement*, GUM 1995 with minor corrections, 2008. Sometimes referred to as "the GUM" or "the ISO GUM."

3.1.15**hysteresis**

The difference between the indications of a measuring instrument when the same value of the quantity measured is reached by increasing or decreasing the quantity.

EXAMPLE Given two 1-kg masses on a scale that has stabilized at 2.000 kg. If one of the masses is removed and then returned, the scale may stabilize at something other than 2.000 kg (probably low), and if another 1-kg mass is put on the scale and then removed, the scale may read something other than 2.000 kg (probably high). The differences (not necessarily equal) are due to hysteresis effects. See related term: **dead band**.

3.1.16**indirect measurement** (adapted from JCGM 200:2008, 2.52, Example 4, Note 1)

A measurement that produces a final result by calculation using results from one or more direct measurements.

EXAMPLE The area of a rectangular sheet of paper is determined from multiplying the results of the direct measurement of the lengths of two adjacent sides of the paper.

3.1.17**instrumental drift** (JCGM 200:2012, 4.21)

Continuous or incremental change over time in indication, due to changes in metrological properties of a measuring instrument.

3.1.18**ISO GUM**

See **GUM**.

3.1.19**mean**

The arithmetic average of a population.

3.1.20**measurand** (JCGM 200:2012, 2.3)

Quantity intended to be measured.

3.1.21**measurement** (JCGM 200:2012, 2.1)

The process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity.

3.1.22**measurement accuracy** (JCGM 200:2012, 2.13)

Closeness of agreement between a measured quantity value and a true quantity value of a measurand.

NOTE Accuracy is a qualitative concept; the quantitative agreement with the true value is expressed as uncertainty.

3.1.23**measurement bias** (JCGM 200:2012, 2.18)

Estimate of a systematic measurement error.

3.1.24**measurement error**

Difference between true and observed values.

3.1.25**measurement method** (JCGM 200:2012, 2.5)

Generic description of a logical organization of operations used in a measurement.

3.1.26**measurement model** (JCGM 200:2012, 2.48)

Mathematical relation among all quantities known to be involved in a measurement.

3.1.27**measurement precision** (JCGM 200:2012, 2.15)

Closeness of agreement between indications or measured quantity values obtained by replicate measurements on the same or similar objects under specified conditions.

3.1.28**measurement process**

Implementation of the measurement method.

3.1.29**measurement repeatability** (JCGM 200:2012, 2.21)

Measurement precision under a set of repeatability conditions of measurement.

3.1.30**measurement reproducibility** (JCGM 200:2012, 2.25)

Measurement precision under reproducibility conditions of measurement.

3.1.31**measurement result** (JCGM 200:2012, 2.9)

Set of quantity values being attributed to a measurand together with any other available relevant information.

3.1.32**measurement uncertainty** (JCGM 200:2012, 2.26)

Non-negative parameter characterizing the dispersion of the quantity values being attributed to a measurand, based on the information used.

3.1.33**outlier**

A value that differs considerably from the typical values in a set.

3.1.34**parameter**

A numerical characteristic of a population.

3.1.35**population** (JCGM 100:2008, C.2.16)

The totality of items under consideration.

3.1.36**precision**

See **measurement precision**.

3.1.37**quantity** (adapted from JCGM 200:2008, 1.1)

Property of a phenomenon, body or substance where the property has a magnitude that can be expressed as a number.

3.1.38**quantity value** (JCGM 200:2012, 1.19)

Number and reference together expressing magnitude of a quantity.

EXAMPLE Length of a given rod: 5.34 m.

3.1.39**random error**

See **random measurement error**.

3.1.40**random measurement error** (JCGM 200:2012, 2.19)

Component of measurement error that in replicate measurements varies in an unpredictable manner.

3.1.41**range**

The difference between the maximum and the minimum values in a set; or a set of all values lying between two given values.

3.1.42**repeatability**

See **measurement repeatability**.

3.1.43**repeatability condition** (JCGM 200:2012, 2.20)

Condition of measurement, out of a set of conditions that includes the same measurement procedure, same operators, same measuring system, same operating conditions and same location, and replicate measurements on the same or similar objects over a short period of time.

3.1.44**reproducibility**

See **measurement reproducibility**.

3.1.45**reproducibility condition** (JCGM 200:2008, 2.24)

Condition of measurement out of a set of conditions that includes different locations, operators, measuring systems, and replicate measurements on the same or similar objects.

3.1.46**result**

See **measurement result**.

3.1.47**sample**

A portion extracted from a population that may or may not contain the constituents in the same proportions as are present in the population.

3.1.48**sensitivity coefficient** (adapted from JCGM 100:2008, 5.1.3)

Numerical description of the change in output, y , given a change in one of the inputs x_i in a function $y = f(x_1, x_2, \dots, x_n)$, where i is a positive integer $\leq n$.

NOTE Technically the sensitivity coefficient is the partial derivative of y with respect to x_i , but in practice the sensitivity coefficient may be more easily approximated algebraically.

3.1.49**stability of a measuring instrument** (JCGM 200:2012, 4.19)

Property of a measuring instrument, whereby its metrological properties remain constant in time.

3.1.50**standard deviation** (JCGM 100:2008, C.2.21)

The positive square root of the variance.

3.1.51**standard measurement uncertainty** (JCGM 200:2012, 2.30)

Measurement uncertainty expressed as a standard deviation.

3.1.52**student's t**

A statistical function that varies in magnitude with degrees of freedom.

3.1.53**systematic error****systematic measurement error** (JCGM 200:2008, 2.17)

Component of measurement error that in replicate measurements remains constant or varies in a predictable manner.

3.1.54**true quantity value** (JCGM 200:2012, 2.11)

Quantity value consistent with the definition of a quantity.

3.1.55**true value**

See **true quantity value**.

3.1.56**Type A evaluation of measurement uncertainty** (JCGM 200:2012, 2.28)

Evaluation of a component of measurement uncertainty by a statistical analysis of measured quantity values obtained under defined measurement conditions.

3.1.57**Type B evaluation of measurement uncertainty** (JCGM 200:2012, 2.29)

Uncertainty determined by means other than a Type A evaluation of measurement uncertainty.

3.1.58**uncertainty**

See **measurement uncertainty**.

3.1.59**variance**

A statistical value that represents the dispersion about the mean within a set of numbers.

3.1.60**verification** (JCGM 200:2012, 2.44)

Provision of objective evidence that a given item fulfills specified requirements.

3.2 Acronyms, Abbreviations, and Symbols

BPV	base prover volume
C_i	The sensitivity coefficient c_i describes how the output y varies with changes in the values of x_i ($i = 1, \dots, n$)
CPL	correction factor for the effect of pressure on a liquid
CPL_f	CPL of the liquid passing through a meter not during a proof
CPL_m	CPL of the liquid passing through a meter during a proof
CPL_p	CPL of the liquid in a prover
CPS	correction factor for the effect of pressure on steel
CTL	correction factor for the effect of temperature on a liquid
CTL_f	CTL of the liquid passing through a meter not during a proof
CTL_m	CTL of the liquid passing through a meter during a proof
CTL_p	CTL of the liquid in a prover
CTS	correction factor for the effect of temperature on steel
dof	degrees of freedom, reference 4.13
k	coverage factor, reference 5.9
GSV	gross standard volume
MF	meter factor

NKF	nominal K-factor
$r_{(i,j)}$	the correlation coefficient between x_i and x_j , where $i \neq j$ and $ r \leq 1$
RTD	resistance temperature detector (also commonly called resistance temperature device)
s	sample standard deviation, an estimate of population standard deviation, reference 4.11
U	expanded uncertainty, reference 5.9
u_i	standard uncertainty of input x_i , reference 4.12
\bar{x}	sample mean, an estimate of population mean, reference 4.10
x_i	the input quantity, is x_i ($i = 1, \dots, n$), a quantity used to determine y through the function $y = f(x_1, x_2, \dots, x_n)$; the input quantities may be viewed as measurands since they too are other quantities
y	the measurand of interest that, in most cases, is determined from n other quantities x_1, x_2, \dots, x_n through the function $y = f(x_1, x_2, \dots, x_n)$
μ	population mean, reference 4.10
σ	population standard deviation, reference 4.11

4 Basic Concepts

4.1 General

This section expands on the definitions as an introduction to Section 5.

4.2 Measurement Method

Measurement is the process of using an instrument to give a numeric indication of a property of some physical object. The result of a measurement is a number and a unit of measurement. Following are three axioms of measurement:

1. A true value exists for this property at the time the measurement is performed.
2. Doubt always exists regarding the accuracy of a measurement result.
3. The arithmetic mean of repeated measurements under repeatability conditions is the best estimate of true value.

The result of a measurement method, hereafter called measurement process, is the determination of the quantity value of a measurand. The measurement process may be direct or indirect. Examples of measurands include pressure, temperature, and volume. The process result is the quantity value; examples include 1452 psig, 86 °F, and 34.7 bbl.

A typical measurement process is made up of multiple components, each of which contributes to the result:

- procedure(s),
- operator(s),

- an environment,
- instrument(s),
- maintenance.

Each component will exhibit variations that contribute to variations in the process result; multiple results from the same measurement process will vary. The potential for many small variations always results in an imperfect process.

4.3 True Quantity Value

Many quantity values exist; each is a result of a measurement process, and they may not be equal. The true quantity value is that which is determined from a perfect measurement process. All measurement processes are imperfect; therefore, the true quantity value is always unknown. The measured quantity value is the result of an imperfect (realistic) measurement process.

4.4 Error

The error is the difference between a measured quantity value and the true quantity value. The true quantity value is unknown; therefore, the error is unknown.

4.4.1 Types of Errors

Measurement error may be viewed as having three components of which two are always present: random error and systematic error.

The third type of error, gross error, such as misapplication of a procedure; a mistake in recording or analyzing data; or using results from equipment that is malfunctioning can introduce significant error in a measurement result. Such blunders must be identified and data from such must be eliminated from any statistical analysis. An example of a blunder would be entering a prover volume of 10.83 bbls into a flow computer when the calibrated volume was reported as 10.38 bbls.

Caution: One way of identifying a blunder is to conduct an “outlier test” which is a mathematical procedure to detect unusual results. See Annex F for more details.

Random error comes about through unpredictable variations of components that influence the measurement. Random errors appear as different results of apparently repeated identical measurements. The variations of these components are not controlled because of ignorance or by a conscious choice (e.g. it's judged to be too expensive). Random error cannot be completely eliminated, but it can be reduced by increasing the number of measurements or by using measurement equipment with greater precision.

Systematic error comes about from an inaccuracy of components that influence the measurement. Systematic errors include imperfect calibration of measurement equipment, imperfect observation methods or interference with measurement system by the environment. An example of a systematic error would be to use a volume of 10.02 gal for a test measure as reported on the calibration certificate but the true value being 9.99 gal. This error will have the same effect on all measurements involving this test measure and, once known, can be eliminated. Systematic error cannot be completely eliminated but can be reduced by performing more frequent calibrations or by using more precise calibration equipment.

The desired outcome of a measurement is a report of the result of the measurement and its associated reliability or uncertainty. With the advent of publishing the *GUM*, the focus on improving measurement shifted from dealing with the types of errors associated with a measurement to determining the impact of each error on the total uncertainty of the measurement. This shift avoided the difficulty of categorizing each identified error and the controversy associated with combining the uncertainties of these two types of errors in favor of

considering each measurement error as a random variable that can be characterized by a probability distribution from which a standard uncertainty can be determined.

Pre-GUM, the terms random and precision as well as the terms systematic and bias were used interchangeably. Precision is a qualitative description of random errors of the mean of a set of measurements quantified by a statistic such as standard deviation. A related term, accuracy, is a qualitative judgment of closeness to True Value. Accuracy cannot be quantified because the True Value is unknowable. It is possible to have any of the four combinations of (in)accuracy and (im)precision as implied in Figure 1 below.

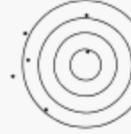
			
Diagram 1	Diagram 2	Diagram 3	Diagram 4
Accurate	Not Accurate	Accurate	Not Accurate
Precise	Precise	Not Precise	Not Precise

Figure 1—Accuracy/Precision Comparisons

If the diagrams above represent a specification, then the decision around using the terms imprecise and inaccurate is in comparison to the specification. Note that the measurement results have no bearing on the “merits” of the specification. For instance, the specification might be a six-inch target which may be reasonable for using a scoped rifle at 25 yd but may be unreasonable for the same rifle at 2000 yd. Additionally, the bias seemingly apparent in diagrams 2 and 4 would be estimated by calculating the central tendency of the results and then calculating the distance between the central tendency and assumed true value of the center of the diagram.

Repeatability is the quantitative description of precision of the data set collected under conditions of repeatability; reproducibility is the quantitative description of the precision of the data set collected under condition of reproducibility (See 3.1.35 and 3.1.37 for definitions). Repeatability and reproducibility are calculated the same way; the difference lies in the conditions under which the set of measurements are made. The calculation is a prediction of the difference between the mean of a set of measurements and the next single measurement made under their respective conditions and is expressed with a confidence interval.

If a systematic error is known to exist, then each measurement with this error should be corrected for the bias. For instance, assume the calibration certificate accompanying an electronic temperature device includes a table of offsets at various temperatures. If the table indicates a bias of -0.1 °F at an indication of 100 °F, then a temperature reading of 100 should be corrected to 100.1 °F. before being used in subsequent calculations or being reported.

4.5 Statistical Basis

This standard assumes a measurement process where all variations (procedure, operator, environment, etc.) are random, are normally distributed, can be observed, and can be described by the measurement model equation:

$$y = f(x_1, x_2, \dots, x_n) \quad (1)$$

where

n is the total number of components used to estimate y ;

x_i represents a contribution to the measurement result, y .

4.6 Uncertainty

Measurement uncertainty defines the estimated range of potential errors. It defines a statistical interval about the measured quantity value within which we are confident to a stated degree the true quantity value lies.

4.7 Confidence Level

The degree of confidence, expressed as a percent, is that the true quantity value lies within the stated uncertainty range. For example, “volume = 500 bbl \pm 1.0 % at a 95 % level of confidence” means that 95 out of every 100 observations are between 495 and 505 bbl. Typically within the petroleum industry, 95 % is used as the level of confidence, and this is what will be used in this standard. Other applications may use other levels of confidence. It is crucial that any statement of a measurement include three components: 1) the measurement; 2) the uncertainty of the measurement; and 3) the confidence level around the measurement. Calculating the confidence level is discussed in 4.14.

4.8 Type A vs Type B Uncertainties

The ISO *GUM* divides the uncertainties of components into two different categories, Type A and Type B, depending how the uncertainties are determined. Type A uncertainties are determined by a statistical analysis on experimental data, and Type B uncertainties, by definition, are uncertainties that are not Type A. Type B analyses are based on judgment and modeling and may include Type A uncertainties within its subparts. There is no inherent quality difference or preference between the two categories. This distinction is used in the calculation of the uncertainty of each component. See the ISO *GUM* for details.

4.9 Representative Samples

A fundamental assumption underlying all statistical analyses performed on a sample of a population is that the sample is representative. A representative sample is an unbiased indication of what the population is like. While the concept is easy to understand, taking a representative sample is very difficult to perform.

For example, assume an analyst wishes to know the meter factor of a meter. The meter factor is determined by calculating the mean (see 4.10) of a number of trial meter factors, each of which are determined through a measurement process known as a meter proof. The true meter factor, μ is impossible to determine because that would require an infinite number of meter proofs; accordingly, a finite set of meter proofs are performed (the sample) and the mean of the resulting trial meter factors is used as the estimate of the true meter factor. These trial factors are representative to the extent that process conditions are perfectly stable during the proof and during the time the meter factor is used.

Another example would be the problem with high speed sampling. The analyst must fully understand the measurement and sample processes to ensure samples taken are truly representative of the total data set with all random error sources having had an opportunity to influence the result.

4.10 Mean

The mean, μ , is a statistical parameter that represents the arithmetic average of a population. If the population is a series of measurements performed on a measurand, then the mean is considered to be the best estimate of the true quantity value and is a Type A component. For infinite populations μ cannot be calculated; for very large populations, μ cannot be practically calculated. In either case a small set of representative samples from

a population are taken and the sample mean, \bar{x} , is used to estimate μ . The general equation for calculating \bar{x} is:

$$\bar{x} = \frac{1}{n}(x_1 + x_2 + x_3 + \dots + x_n) \quad (2)$$

where

x_i are the elements of the sample;

n is the number of elements in the sample.

EXAMPLE Assume a set of trial meter factors: 0.9765, 0.9752, 0.9741, 0.9743, and 0.9750. Using Equation (2), the calculated sample mean (\bar{x}) is 0.97502.

4.11 Standard Deviation

The population standard deviation, σ , is a statistical value that represents the dispersion about the mean within a set of numbers, called a population. For infinite populations σ cannot be calculated; for very large populations, σ cannot be practically calculated. In either case, a small set of representative samples from a population are taken and the sample standard deviation, s , is used to estimate σ . This makes s a Type A component.

EXAMPLE 1 The standard deviation of a set of five trial meter factors, s , is used to approximate the population standard deviation, σ , that could only be calculated from an infinite number of proofs.

EXAMPLE 2 The standard deviation of the thickness of a lot of one million washers is estimated by measuring the thickness of a representative sample of 100 washers and calculating s . The general equation for calculating s is:

$$s^2 = \sum_{i=1}^n \frac{(x_i - \bar{x})^2}{n-1} \quad (3)$$

where

x_i are the elements of the sample;

n is the number of elements in the sample.

Note that the certainty in the mean as being the true value of a measurand varies inversely with the standard deviation of the set of measurements; that is, the larger the standard deviation, the less certain an analyst can be that the mean is the true value. An example is shown below from meter proving.

Table 1— Data with Same Mean but Different SDs

	Case 1	Case 2
trial 1	1.0001	1.0002
trial 2	1.0002	0.9998
trial 3	0.9999	0.9900
trial 4	1.0002	1.0005
trial 5	1.0001	1.0100
Mean	1.0001	1.0001
SD, s	0.00012	0.00708

In this extreme example, the means of both sets are equal but the standard deviations are quite different. One would be very confident in the Case 1 mean but not at all in the Case 2 mean. Another way to indicate confidence is to predict what the trial 6 factor would be in each case. The best estimate is the mean, but again, one would be very confident in the Case 1 mean but not all in the Case 2 mean.

4.12 Standard Uncertainty

All contributing uncertainties shall be expressed at the same confidence level so they can be combined. This is done by converting them into standard uncertainties. A standard uncertainty is a margin whose size can be thought of as “plus or minus one standard deviation.” The standard uncertainty tells us about the uncertainty of an average as well as the spread of values, and converting all uncertainties to standard uncertainties makes them comparable. A standard uncertainty is usually shown by the symbol $u(x)$, which is interpreted as the standard uncertainty of x .

The estimated standard uncertainty of the mean of a set of measurements (also called the standard deviation of the mean or the standard error of the mean), a Type A component, is calculated from:

$$u(\bar{x}) = \frac{s}{\sqrt{n}} \quad (4)$$

where

\bar{x} is the mean from Equation (2);

s is the sample standard deviation from Equation (3);

n is the number of measurements in the set.

NOTE Equation (4) yields standard uncertainty in engineering units. Depending on the units of the sensitivity coefficient of x_i , (see 5.6), the analyst may have to convert the standard uncertainty to a relative standard uncertainty in units of percent by multiplying $u(\bar{x})$ by $100\%/\bar{x}$.

The user is cautioned that the period of time over which the samples are taken is important. A shorter time interval will represent repeatability and a longer period of time will also include reproducibility. Depending on the timing of the readings, Equation (4) gives the standard uncertainty of a mean due to repeatability or reproducibility only and does not include other possible sources of uncertainty. See the definitions of reproducibility and repeatability.

Calculating the standard uncertainty for Type B components is discussed in 5.5.5.3.

4.13 Degrees of Freedom

In general, degrees of freedom (dof) is an indication of how much independent information was used in the determination of a statistic or a parameter. For this standard, dof is the number of independent measurements used to determine the standard uncertainty of the mean of these measurements; dof has meaning only for Type A components. Dof are generally considered infinite for a Type B component.

For example, suppose an analyst had the following five meter factor measurements: 0.9765, 0.9752, 0.9741, 0.9743, and 0.9750. Using Equation (2), the mean is 0.97502 and the dof for calculating the mean is five because there are five independent pieces of information that are used for the calculation. The calculated standard deviation using Equation (3) is 0.00095. This statistic has $5 - 1$ dof because the equation uses the mean to calculate the standard deviation and the mean is not independent of the meter factors. By using the mean we have lost 1 dof. This standard uses dof to determine the confidence level.

4.14 Determining Confidence Interval

An analyst will be more confident in a conclusion based on a larger sample as compared to a smaller sample. This confidence is quantified by using a factor called Student's t , which varies in magnitude with confidence level and dof as shown in Table 1. The confidence in a mean of a sample is quantified by

$$\bar{x} \pm t_{(\text{confidence level, dof})} u \quad (5)$$

where

\bar{x} , is from Equation (2);

$t_{(\text{confidence level, dof})}$ is from Table 1;

u , is from Equation (4).

For the example in 4.13:

n = 5 (sample size);

\bar{x} = 0.97502 (mean);

s = 0.00095 (estimate of standard deviation);

$u = \frac{s}{\sqrt{n}} = \frac{0.00095}{\sqrt{5}}$

dof = 4 (degrees of freedom);

$t_{(\text{confidence level, dof})} = t_{95\%,4} = 2.7764$ (from Table 1);

the 95 % confidence interval is $0.97502 \pm \frac{0.00095}{\sqrt{5}} \times 2.7764$ or (0.9738, 0.9762).

Table 2—Student t Factors for Individual Measurements at Various Confidence Levels

dof	t_{68} (1s)	t_{90}	t_{95} (2s)	t_{99} (3s)
1	1.8190	6.3138	12.7062	63.6567
2	1.3116	2.9200	4.3027	9.9248
3	1.1889	2.3534	3.1824	5.8409
4	1.1344	2.1318	2.7764	4.6041
5	1.1037	2.0150	2.5706	4.0321
10	1.0464	1.8125	2.2281	3.1693
25	1.0146	1.7081	2.0595	2.7874
50	1.0044	1.6759	2.0086	2.6778
100	0.9994	1.6602	1.9840	2.6259
∞	0.9945	1.6449	1.9600	2.5758

NOTE The values in this table can be duplicated by using appropriate inputs into the double-sided Student t distribution function found in an electronic spreadsheet.

In another example, suppose Analyst A takes an average of 5 trial meter factors to represent the performance of a meter compared to Analyst B, who takes an average of 3 trial factors. Let us also suppose that the mean of both analysts is 0.9800 and the standard deviation for both analysts is 0.001.

Analyst A has $5 - 1 = 4$ dof, which yields a 95 % confidence interval of $0.98 \pm 2.7764 \frac{0.001}{\sqrt{5}}$ or (0.9788, 0.9812).

Analyst B has $3 - 1 = 2$ dof, which yields a 95 % confidence level of $0.98 \pm 4.3027 \frac{0.001}{\sqrt{3}}$ or (0.9775, 0.9824).

Notice that Analyst B's interval is twice as wide as Analyst A's interval. This is due to not having the same amount of data, which leads to fewer dof and, therefore, less certainty that Analyst B's mean represents the true performance of the meter.

If an uncertainty measure is used from a Type B situation, this value should be thought to come from one of the following distributions: normal, triangular, or rectangular. Degrees of freedom are not used in these cases; however, it should be accurately determined what distribution is being used so the proper standard uncertainty calculation is made. If no information is available, then by custom, a normal distribution with infinite dof is assumed.

If the analyst has limited data, the dof need to be calculated accurately because an error will have a significant impact on calculations concerning confidence levels. Table 1 shows t-values at the 68 %, 90 %, 95 %, and 99 % confidence level with associated dof. This will allow the analyst to translate from one commonly used confidence level to another, if needed. Given a normal distribution, the 95 % confidence level interval width is

$$\pm 1.96 \frac{\sigma}{\sqrt{n}} .$$

Notice as dof approaches infinity, the t-distribution converges to the normal distribution. The width of the corresponding interval will be $\pm t_{(95\%, dof)} \times \frac{s}{\sqrt{n}}$ for a sample of the same population.

4.15 Correlation Coefficient

Correlation is the degree of relationship between two variables. As used in this standard, the correlation coefficient, $r(x_i, z_i)$, quantifies the strength and direction of the linear relationship between the elements in two n -sized sets of measurements, x_i , and z_i , and is defined by the following equation:

$$r(x_i, z_i) = \frac{\sum_{i=1}^n (x_i - \bar{x})(z_i - \bar{z})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (z_i - \bar{z})^2}} \quad (6)$$

The correlation coefficient is a number between -1 and $+1$. The correlation coefficient is symmetric so $r(x_i, z_i) = r(z_i, x_i)$. This standard will denote the correlation coefficient by $r(x, z)$, $r_{(i,j)}$, or by r if the meaning from context is clear.

- $r = -1$ means a scatter plot of the two sets of measurements form a line with negative slope; that is, they have strong negative correlation.
- $-1 \leq r < 0$ means that as r goes from -1 to 0 , the scatter plot less closely resembles a line with negative slope; that is, the strength of negative correlation decreases.
- $r = 0$ means there is no linear relationship at all.

- $0 < r \leq 1$ means that as r goes from 0 to 1, the scatter plot more closely resembles a line with positive slope; that is, the strength of positive correlation increases.
- $r = +1$ means a scatter plot of the two set of measurements form a line with positive slope; that is, they have strong positive correlation.

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4.17 Combining Uncertainties

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$$y = f(x_1, x_2, \dots, x_n) \quad (1)$$

For reasons explained in Annex E of the ISO *GUM*, the standard uncertainty of y is given by:

$$u_y^2 = \sum_{i=1}^n c_i^2 u_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n r_{(i,j)} c_j c_i u_j u_i \quad (7)$$

where

- u_y is the standard uncertainty of y ;
- n is the number of elements in Equation (1);
- c_i is the sensitivity coefficient of x_i to y , this is discussed in detail in 5.6;
- u_i is the standard uncertainty of x_i , this is discussed in detail in 5.5;
- $r_{(i,j)}$ is the correlation coefficient for x_i to x_j where $i \neq j$, this is discussed in detail in 5.7.

Note that the first term on the right-hand side (RHS) of Equation (7) will always be positive in sign. The second RHS term can be positive or negative depending on the signs of $r_{(i,j)}$, c_i , and c_j . The effect of correlation can increase or decrease the uncertainty depending on the sign of this term. See Annex B.2 for an example.

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where

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- n is the number of elements in Equation (1);
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$$u_y^2 = \sum_{i=1}^n c_i^2 u_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n r_{(i,j)} c_j c_i u_j u_i \quad (7)$$

where

- u_y is the standard uncertainty of y ;
- n is the number of elements in Equation (1);
- c_i is the sensitivity coefficient of x_i to y , this is discussed in detail in 5.6;
- u_i is the standard uncertainty of x_i , this is discussed in detail in 5.5;
- $r_{(i,j)}$ is the correlation coefficient for x_i to x_j where $i \neq j$, this is discussed in detail in 5.7.

Note that the first term on the right-hand side (RHS) of Equation (7) will always be positive in sign. The second RHS term can be positive or negative depending on the signs of $r_{(i,j)}$, c_i , and c_j . The effect of correlation can increase or decrease the uncertainty depending on the sign of this term. See Annex B.2 for an example.

NOTE 1 It is imperative that the units of all the c_i and u_i be in either absolute terms or in relative terms. How to convert one form to the other is discussed in 5.5 and 5.6.

NOTE 2 In order to calculate the correlation coefficient for use in Equation (7), the analyst needs two vectors of data, one from x and the other from z . For the typical analyst of this standard, most of the components used in Equation (8) will come from a Type B evaluation of uncertainty, which means no data exist to calculate $r_{(i,j)}$. Accordingly, this standard assumes r equals 1 or -1 for correlated Type B components depending on the direction of the relationship, and if the two components have a very weak or no relationship, r is assumed to equal 0.

Partially correlated or weakly correlated uncertainties are commonly caused by components that have not been broken down into their elemental uncertainties but represent a combined uncertainty component. For example the uncertainty of CTL is a combination of the temperature measurement uncertainty, the liquid

- $0 < r \leq 1$ means that as r goes from 0 to 1, the scatter plot more closely resembles a line with positive slope; that is, the strength of positive correlation increases.
- $r = +1$ means a scatter plot of the two set of measurements form a line with positive slope; that is, they have strong positive correlation.

4.16 Independent and Dependent Variables

Two variables are independent if knowing information about one in no way helps explain or understand the other; that is, no relationship exists between the two variables. Two variables are dependent if they are not independent, and correlation refers to the measure of departure the two random variables are from being independent. If two variables are independent, then they have a correlation coefficient of zero. The concepts of dependence and correlation become important when combining uncertainties.

4.17 Combining Uncertainties

4.17.1 General

Assume the measurement model shown by Equation (7) reproduced below:

$$y = f(x_1, x_2, \dots, x_n) \quad (1)$$

For reasons explained in Annex E of the ISO *GUM*, the standard uncertainty of y is given by:

$$u_y^2 = \sum_{i=1}^n c_i^2 u_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n r_{(i,j)} c_j c_i u_j u_i \quad (7)$$

where

- u_y is the standard uncertainty of y ;
- n is the number of elements in Equation (1);
- c_i is the sensitivity coefficient of x_i to y , this is discussed in detail in 5.6;
- u_i is the standard uncertainty of x_i , this is discussed in detail in 5.5;
- $r_{(i,j)}$ is the correlation coefficient for x_i to x_j where $i \neq j$, this is discussed in detail in 5.7.

Note that the first term on the right-hand side (RHS) of Equation (7) will always be positive in sign. The second RHS term can be positive or negative depending on the signs of $r_{(i,j)}$, c_i , and c_j . The effect of correlation can increase or decrease the uncertainty depending on the sign of this term. See Annex B.2 for an example.

NOTE 1 It is imperative that the units of all the c_i and u_i be in either absolute terms or in relative terms. How to convert one form to the other is discussed in 5.5 and 5.6.

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Partially correlated or weakly correlated uncertainties are commonly caused by components that have not been broken down into their elemental uncertainties but represent a combined uncertainty component. For example the uncertainty of CTL is a combination of the temperature measurement uncertainty, the liquid